MA 398 Homework 3: Not a Length-y one.

In class we defined infimum and supremum of subsets of $\mathbb{R}$. The first few problems are intended to reinforce those ideas. The other problems on this homework will get you ready for discussing continuous functions.

1. READING

- Schwartz: Sections 1.1 - 1.5
- Bonahon: Sections 1.1, 1.2, 1.4

The reading from Schwartz's book gives you an overview of some of what we'll be doing. The reading from Bonahon's book introduces euclidean geometry in the way we'll need it for the rest of the course.

2. HUTS

These problems are intended to give you some practice with basic concepts. They will often involve calculation, rarely involve new ideas, and won't be graded. However, your answers will be collected!

(1) Do Exercise 1.1 on page 7 of Bonahon.

(2) Do Exercise 1.10 on page 9 of Bonahon.

3. HOUSES

These problems are intended to require more thought and less calculation.

(1) Let X be a set and let $\delta\colon X \times X \to [0,\infty) \subset \mathbb{R}$ be a function such that for all $x, y \in X$, $\delta(x,y) = \delta(y,x)$. Let $C\colon x_0, x_1, \ldots, x_n$ be a chain in X joining x to y. Recall that the length $\delta(C)$ of C is defined to be $\sum_{i=1}^{n} \delta(x_{i-1}, x_i)$. Define the grasshopper distance from x to y to be $d(x,y) = \inf_C \delta(C)$. Prove that d satisfies the triangle inequality.

(2) Do exercise 1.3 on page 8 of Bonahon. You may use 1.2 without proof.