MA 398 Homework 14: Putting the silly back into cylinder.

1. Huts

(1) Read Schwartz Sections 12.1 - 12.3 and Bonahon Section 4.5. Write a few sentences comparing Schwartz's approach to Bonahon's approach. Do they both include all the details? If not, what gaps are there – do you think they're big or small? Are they proving the same thing? Whose approach is easier and why doesn't the other do it that way?

2. Houses

(1) In this problem your task is to think about ways of putting euclidean and hyperbolic structures on an infinite cylinder. (In what follows we will call a cylinder any surface homeomorphic to the result of revolving the line $x = 1$ around the y-axis.

(a) In $\mathbb{E}^2$, let P be the polygon bounded by the lines $x = -1$ and $x = 1$. Let $\phi_r(-1, y) = (1, y + r)$. and let $\overline{P}_r$ be the quotient space with the grasshopper metric. Recall that you have thought about these surfaces on an earlier homework assignment. Prove (by referencing appropriate theorems) that $\overline{P}_r$ is a euclidean surface (i.e. each point has a small neighborhood isometric to a disc in $\mathbb{E}^2$.)

(b) In $\mathbb{E}^2$, let X be the polygon bounded by the half-lines

$$\begin{aligned} y &= -x - 1 \text{ for } x \leq -1 \\ y &= x + 1 \text{ for } x \leq -1 \\ y &= x - 1 \text{ for } x \geq 1 \\ y &= -x + 1 \text{ for } x \geq 1 \end{aligned}$$

Specify edge gluings so that the quotient space $\overline{X}$ is homeomorphic to a cylinder. Decide if $\overline{X}$ is locally euclidean and why or why not. Describe in words what the geometry is: what do geodesics look like? what are the shortest closed geodesics?

(c) Specify a polygon Y in $\mathbb{H}^2$ and edge gluings so that $\overline{Y}$ is homeomorphic to a cylinder and so that $\overline{Y}$ is a hyperbolic surface.

Reference specific theorems to verify that it is a hyperbolic surface. Go on to create infinitely many non-isometric hyperbolic cylinders.

(d) Explain why you cannot give the cylinder a spherical geometry in a way similar to what we did for euclidean and hyperbolic geometries.