MA 398 Homework 13: Gluing Polygons, cont.

These problems are intended to help improve your intuition for gluing polygons in euclidean, spherical and hyperbolic space.

1. Huts

(1) Do this task twice and create different examples. Draw an unbounded polygon in $\mathbb{E}^2$ (with one or more sides consisting of lines or half lines), specify gluing maps to glue the edges together in pairs. Sketch a picture of the resulting object up to homeomorphism. Draw some ε balls centered at various points on the glued-up polygon. Some of the ε-balls should intersect the edges of the polygon and wrap around.

(2) Draw a, possibly disconnected, polygon on S^2 with edges consisting of portions of great circles and specify gluing maps so that the resulting surface is homeomorphic to the projective plane. What do the angles sum to around the vertices?

2. Houses

Consider the disc model of $\mathbb{H}^2$. Let T be a hyperbolic triangle, that is a triangle with geodesic edges.

(1) If the vertices of T are all on $S^1_\infty = \partial\mathbb{H}^2$, explain why the angle sum of T is 0 and why this means the area is π. (You may appeal to the result proven in class.)

(2) If the vertices of T are all near the origin, explain why this means that the angle sum is is very close to π.

(3) Use a continuity argument to prove that there is an equiangular triangle such that the angle sum is any particular number in the interval $(0,\pi)$.

(4) Now do the same thing but for an octogon with geodesic sides. In this case you should be able to prove that there is a regular hyperbolic octogon such that the angle sum is any particular number in the interval $(0,6\pi)$. In particular, there is a regular octogon with angle sum equal to 2π and each angle equal to $\pi/4$. Speculate on how we can use this octogon to give the genus 2 surface $T^2 \# T^2$ a metric that is locally isometric to the hyperbolic metric.