MA 262: Project Topics

At the end of the semester, you will be giving an oral presentation and turning in a written document pertaining to material related to vector calculus. Below is a list of possible topics. Please rank your top 3 choices. I will place you into groups of 3 - 4 people and will assign you a project based on your rankings. If you would like any more information about these topics, please ask.

(a) _____ Improper Applications of Green's Theorem

Green's Theorem can be used to evaluate improper integrals that show up in 1-variable Calculus (for example, $\int_0^\infty (\sin x)/x\,dx$.) For this project, you will read one or two very short articles which show how this can be done.

(b) _____ Total Curvature of a Knot

At the age of 19, John Milnor (who was recently awarded the Abel Prize) proved a theorem now called the Fary-Milnor theorem (which had been proven slightly earlier by István Fáry). In this project you will define the notion of "total curvature" of a curve in and discuss Milnor's proof that any knot with a total curvature of no more than 4π is unknotted. Your sources will be articles by Milnor and Spivak.

(c) _____ Divergence and Gauss' theorem

In class we won't have a lot of time to discuss this generalization of the "planar divergence theorem", so in this project you'll develop these ideas as well as the related notion of "vector potential". The fundamental question to explore is: Suppose that $\mathbf{F}$ is a vector field in $\mathbb{R}^3$ with $\operatorname{div}\mathbf{F} = \mathbf{0}$. Is $\mathbf{F} = \operatorname{curl}\mathbf{G}$ for some vector field $\mathbf{G}$?

(d) _____ (Co)homology theory

Vector fields are objects that (among other things) record the velocity and direction of fluid flow. Scalar fields are objects (that among other things) record the temperature at each point in space. In our course, we study three basic operations on scalar fields and vector fields: gradient, curl, and divergence. It turns out that the relationship between these three operations (actually, they are all "derivatives" of some sort) provide the foundation for a very powerful and interesting theory called "(co)homology theory". In this project, using notes written by your professor, you'll discover how the material of our course fits into the larger theory.

(Continued on reverse)

(e) _____ The Gravity of Hades

In this project you'll read an article that adds nuance to our discussion of gravity. The article uses vector calculus, some physics, and some famous literature to understand what sort of gravitational forces Opheus experienced on his trip to rescue Eurydice.

(f) _____ Maxwell's Equations

Maxwell's equations are fundamental to the theory of electromagnetism. In this project, you will explain what Maxwell's Equations say and will sketch their proof.

(g) _____ Swimming in Molasses

What sort of physics and math apply if you tried to swim in molasses? Whatever it is, its the sort of physics and math that applies when *E. coli* swims through water. In this project, you'll read an essay that investigates what sort of mechanisms enable creatures to swim through very viscous fluids.

(g) _____ Green's identity and vortex atoms

One of the most important consequences of Green's theorem is called "Green's 2nd identity". In this project, you will read a paper that discusses Green's identity and shows how 19th century physicists applied it to the study of "smoke rings" and multiply-connected spaces. Your project will focus on explaining the basics of the mathematics and outlining the history surrounding the identity.

(h) _____ Your own idea

You may propose your own project which must be related to this course. Such a project may take the form of original research. For example, you may want to try to write down a rigorous proof of Green's Theorem (in particular, how does the limiting argument work?)