MA 262: Review of material through Exam 2 **Name:**____________________________

(1) Derivatives:

(a) Understand and be able to calculate the derivative as a matrix
(b) Understand the definition of C^1 and of differentiable
(c) Be able to find the equation for the affine approximation to a function at a point
(d) Know and be able to use the chain rule

(2) Parameterized Curves

(a) Know parameterizations for common curves (circles, straight lines, graphs of functions)
(b) Understand and be able to use tangent space coordinates to find the parameterizations of complicated curves (epicycles, cycloids, etc.)
(c) Understand what the derivative of a parameterized curve measures
(d) Be able to reparameterize a curve with an orientation preserving or reversing change of coordinates function.
(e) Understand the difference between intrinsic and extrinsic properties of curves
(f) Be able to write down an integral representing the length of a parameterized curve.

(3) The geometry of parameterized curves

(a) Be able (in practice and principle) to reparameterize a curve by arclength.
(b) Be able to prove that the unit tangent vector **T** is intrinsic to oriented curves
(c) Be able to calculate the moving frame **T**, **N**, and **B** (although **B** won't be on the exam.)
(d) Be able to calculate curvature $\kappa(t)$.
(e) Understand the idea of tangential and normal components to acceleration.
(f) Be able to prove that in a 2-body system consisting of a planet and a sun, the planet's orbit will lie in a plane.

(4) Line Integrals

(a) Know that if f is a scalar field and if $\mathbf{x}\colon [a,b] \to \mathbb{R}^n$ is a path then
$$\int_{\mathbf{x}} f\,ds = \int_a^b f(\mathbf{x}(t))||\mathbf{x}'(t)||\,dt.$$

(b) Know that if $\mathbf{F}$ is a vector field and if $\mathbf{x}\colon [a,b] \to \mathbb{R}^n$ is a path then
$$\int_{\mathbf{x}} \mathbf{F}(\mathbf{x}(t)) \cdot \mathbf{x}'(t)\,dt.$$

(c) Understand what the path integral of a vector field measures (work, circulation, etc.) and why.

(5) Vector Fields

(a) Be able to draw a picture of a given vector field
(b) Know the concept of "flow line" and be able to work simple examples
(c) Understand what curl measures and be able to calculate it.
(d) Understand what divergence measures and be able to calculate it.
(e) Know what a conservative/gradient field is and be able to find potential functions for simple examples
(f) Know the basic idea for why conservative vector fields don't have closed up flow lines
(g) Be able to prove that conservative vector fields have path independent line integrals.

(6) Green's Theorem

(a) Know the precise statement of Green's Theorem
(b) Be able to calculate the integrals appearing on both sides of the equality in Green's theorem (that is: "verify" Green's theorem for particular examples.)
(c) Be able to use the integral on one side of Green's theorem to calculate the integral on the other side (that is: "use" Green's theorem for particular examples.)
(d) Use Green's theorem to find areas enclosed by curves
(e) Use Green's theorem to relate the line integral of a vector field around one curve to the line integral of the vector field around a different curve
(f) Be able to explain why Green's theorem is true by identifying the important features of the proof and how they fit together. In

particular, be sure to recall how the mean value theorems for derivatives and integrals are relevant.

(7) Conservative Vector Fields

(a) Be able to apply the theory of conservative vector fields to calculate the work done in moving a particle through the electric field generated by a charged wire.
(b) Be able to prove that if a conservative vector field has path independent line integrals then it is conservative.
(c) Be able to prove that on a simply connected domain if a vector field has zero curl then it is conservative. (Poincaré's theorem)
(d) Use curl to determine whether or not a vector field is conservative
(e) State Poincaré's theorem
(f) Be able to give an example of a vector field having curl zero on its domain which is not a conservative vector field. Understand the relationship between this example and Poincaré's theorem

(8) Planar Divergence Theorem

(a) Know and be able to explain the statement of the planar divergence theorem
(b) Be able to both "verify" and "use" the planar divergence theorem
(c) Be able to use the planar divergence theorem to compare the flux of a vector field through different curves.

(9) Surface Integrals and Stokes' theorem

(a) Know the definitions of "surface" in both the topological and calculus senses
(b) Be able to write down equations for standard parameterizations of surfaces
(c) Be able to match a parameterization with the image of a surface.
(d) Know what a normal orientation for a surface is.
(e) Be able to determine from the equation whether a parameterized surface is smooth and orientable.
(f) Know the definition of surface integral of a scalar field and vector field and be able to do calculations.
(g) Know the statement of Stokes' theorem.