MA 262	Vector Calculus	Spring 2012
Section A	MWF 11 - 11:50	Mudd 405

Professor:	Scott Taylor
Office Hours:	MWF 1 - 3. Tues 11 - 1 *and by appointment!*
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Prerequisites:	MA 122 or permission of instructor
Text:	Colley, *Vector Calculus* 3/e, Pearson. (Note: This is not the current edition!)

The Course:

MA 262, building off the content of MA 122, develops the calculus of functions in all dimensions. The main goal of the course is to relate the concepts of "derivative" and "integral" in all dimensions by generalizing the Fundamental Theorem of Calculus. Along the way you will become adept at using the language and notation of vectors to express key ideas in geometry and physics.

(*A word on course number:* MA 262 was formerly labelled MA 302. The content of the course has not changed at all, it was always a 200-level course. The number was changed to reflect the fact that it has only 100-level prerequisites.)

Objectives for increasing mathematical maturity:

- Develop an intuition for higher dimensions by comparing high dimensional geometry and calculus to low dimensional geometry and calculus.
- Use vectors and vector fields to solve problems in geometry and physics.
- Use geometric and physical intuition to understand mathematical objects.
- Increase understanding of the relationship between definitions, examples, theorems, and proofs in mathematics.
- Begin to explore the intrinsic nature of many geometric definitions (eg. curvature).
- Begin to explore the relationship between geometry and topology by discussing notions such as orientation and path-independence.

- Understand the relationship between Introductory Calculus and Vector Calculus
- Engage in significant self-teaching of mathematics.
- Effectively communicate mathematics.

Major Course Content Objectives

- Use linear algebra to understand the definition and behaviour of the derivatives of functions $f\colon \mathbb{R}^n \to \mathbb{R}^m$.
- Use derivatives to understand intrinsic properties of curves in $\mathbb{R}^2$ and $\mathbb{R}^3$. These include arc length, curvature, and unit tangent vectors.
- Use the calculus of space curves to model various physical situations
- Understand the mathematical and physical significance of the gradient, divergence, and curl of a vector field.
- Understand the mathematical and physical significance of path integrals of scalar and vector fields.
- Understand the mathematical and physical significance of surface integrals of scalar and vector fields.
- Understand the statements of Green's, Gauss', and Stokes' theorems.
- Understand the main ideas in the proof of Green's theorem.
- Understand the concepts of conservative vector fields and their potential functions.

Attendance: I value your involvement in the class, therefore class attendance is mandatory. Absence for official Colby activities requires prior approval. Absence for religious reasons will be considered excused if the policy in the college catalogue is followed. I reserve the right to take attendance. More than 3 recorded unexcused absences will result in the reduction by 1/3 of the final course grade. Excessive tardiness or early departure may also result in such a reduction.

Computing Resources: On quizzes and exams you may use a graphing calculator. The point of an exam is to test for understanding – such understanding must come through clearly in your answers.

You will be required to use the software to complete homework and projects throughout the semester. *Mathematica* is available on many computers at Colby, including the computers in Mudd 415 and Olin 323. From Colby's fileserver you may also download and install *Mathematica* on your personal computer for use while on campus. Details will be distributed later.

Upon occasion you may also wish to make use of two other free pieces of software:

- *WolframAlpha* (wolframalpha.com) is a web-based computational knowledge engine. It can compute most of the integrals and evaluate most of the limits which we will encounter in the course. It can be a bit tricky to use, so you will need think about whatever response it gives you. Another similar tool from the Wolfram people is available at integrals.wolfram.com
- *Grapher* is software which comes bundled with every modern Macintosh. (It can be found under Utilities). It is very easy to use and can draw almost every imaginable type of 2 and 3 dimensional graph. (Including solutions to differential equations and vector fields.) I highly recommend it – it is particularly easy to use.

You are allowed to use software to solve (almost) any integral you encounter in this course. You will be required to use *Mathematica* or *Grapher* to draw paths, surfaces, and vector fields.

Academic Honesty: All work in this course must be your own and you should always be prepared to explain or defend it. You are encouraged to work with other students on the homework assignments and review exercises, but you may not copy another student's work. On exams and quizzes you are not allowed to confer with anybody else, use any notes, books, or online resources, unless otherwise permitted. Calculators are permitted. Please read the section on Academic Honesty in the catalogue (pg 43).

Evaluation: The numerical course grade will be a weighted average of the cumulative grades with weightings as follows:

25%	Weekly Homework	5%	Mathematical Engagement
5%	Term Project	10/20%	Exam 1
10%	Quizzes	10/20%	Exam 2
20%	Final Exam.	5%	Review Exercises

The maximum of your scores for Exams 1 and 2 will count at 20% of your grade. The minimum of your scores on Exams 1 and 2 will count as 10% of your grade.

Caveat: earning less than 50% on each of the three exams will result in a grade of "F" for the course.

Course letter grades will be assigned (subject to the above caveat) according to the following scale. Any curve will be determined at the end of the course

according to the discretion of the instructor. An A+ may be awarded to students who do exceptionally well and who demonstrate an exceptional interest in the course.

93 – 100 % A	90 – 93 % A-	87 – 90 % B+	83 – 87% B
80 – 83 % B-	77 – 80 % C+	73 – 77 % C	70 – 73% C-
67 – 70 % D+	63 – 67 % D	60 – 63 % D-	below 60% F

Homework: Homework is probably the most important part of this course – it's when you get to put into practice the concepts you've played with during class. Some of the homework questions may require you to explore some topic which we didn't discuss in class. The purpose of such questions is to help you develop the ability to read and learn mathematics on your own. If you go into a mathematical or scientific career, there will undoubtedly be times when you need to teach yourself some mathematics. If, however, all the homework problems fall into this category, you should check to make sure that you are working on the correct assignment.

Weekly homework will generally be due on Wednesdays and will always be posted on the course webpage. You are responsible for checking the webpage. If no homework assignment is posted, you should refresh the webpage on your browser and, if that doesn't work, email me to let me know. In the special circumstance that there is no new homework, the webpage will make note of that. If you will not be in class on the day that homework is due you should arrange to turn it in at my office or to have a friend bring it to class. Late homework may be penalized.

You should start the homework early, some problems will require multiple attempts and careful thought. If you are having substantial difficulty with a particular problem or the entire homework set you should email me or come to office hours. I am eager to help you!

You are encouraged to work with a partner on the homework, but all work should be your own. In other words, you may discuss particular problems but you may not copy someone else's solution. Doing so violates academic honesty. As the course progresses, the question of how much work to show will arise. I encourage you to use common sense. If the work pertains to concepts discussed in class or in the reading, you should show it. If the work requires substantial effort and thought, you should show it. If the work is simply evaluating an integral using methods learned in MA 121, you do not need to show it. If the work is simply elementary algebra, you do not need to show it (although showing it may help the grader follow your work). You may even use *Mathematica* or other software to evaluate the 1-variable integrals or derivatives that arise.

In general, your work is your answer. It is possible for someone to obtain a correct answer but to not receive full credit because their work is incorrect. Conversely, (almost entirely) correct work with an incorrect answer may receive full credit.

Homework must be very neat. This means: no messy scratchwork, no cramped writing, no huge eraser marks. Multiple pages should be stapled and the problems should be in order with section and problem number clearly indicated. If these guidelines are not followed you may be penalized. If you are incapable of writing neatly, you should type your solutions. LaTex is the most popular mathematical typesetting software, but you may also use programs like *Scientific Word* or *Mathematica*.

You will occasionally have a reading assignment due in class. These assignments will require that you do a certain amount of reading and answer one or two fairly easy questions. Usually the reading will cover material we have not yet or will not discuss in class. I will always assume that you have done the reading by the time that it is due and will not regurgitate in class material you have read on your own.

Quizzes: You will be given a quiz almost every Friday. If you are absent on the day a quiz is given, it may be made up within one week of when it was given. You are responsible for requesting a make-up quiz. The lowest quiz score will be dropped from the computation of the course grade.

Mathematical Engagement: Half of your "Mathematical Engagement" grade will be based on your class participation. I expect that you will volunteer answers to questions posed to the class as a whole and that you will be prepared to make a contribution to the discussion if specifically called upon. In the event, that I call on you specifically you do not necessarily need to be able to answer the question I ask, but you should be prepared to give a partial answer, an intelligent guess, or to ask a relevant question. The other half of your "Mathematical Engagement" grade will be based on completing assigned readings as well as two of the following activities designed to have you encounter mathematics outside the classroom. You can complete an activity in one of two ways:

(1) Attend a Mathematics/Statistics Colloquium and write a page summarizing and reacting to the lecture.
(2) Read an article on reserve in the Library (see handout) and write a page summarizing and reacting to the article.

Projects: In a group of 2 - 4 people you will complete a substantial project. The content of the project will be related to applications of course material

to other scientific disciplines or to other mathematical ideas. The projects will require that you learn the relevant background, extract the most important points, write a summary of your findings and present your findings to the class during the final week of the course. More details will be distributed later. Successful completion of the project demonstrates your ability to self-teach mathematics and to communicate it effectively to your classmates.

Exams: There will be two in-class exams and a final exam. Each exam is cumulative, although the final exam is "more cumulative". Exams will be designed to test your understanding of the course material, not just your computational abilities. You must understand, and communicate your understanding of the material. Computers, textbooks, notes, and other people may not be used on the exam. The in-class exams will be given on **Friday, March 9** and **Friday, April 20**. The final exam is currently scheduled for exam period **8** on **Friday, May 11** at **1:30 PM**. It may not be rescheduled for personal convenience (including airline reservations).

Review Exercises: I will occasionally give review exercises which ask you to review material typically covered in MA 121 or MA 122. These will usually be due the class period after I assigned them and are in addition to any other homework. It may be that a review exercise covers material you have not actually been taught before. In that case you are expected to either teach it to yourself or to see the professor for help.

How to succeed in this course:

Vector Calculus is an intense 200-level mathematics course. The course, therefore, expects a level of sophistication and mathematical maturity beyond that expected by introductory calculus courses. In particular: you are responsible for your own learning and the professor is responsible for assisting you in your quest. In order to truly understand vector calculus you need to be willing to do computations and you need to be willing to wrestle with ideas. The following advice is meant to provide suggestions on get more out of the class and to improve your grades:

(1) *Do some math everyday.* Really.

Learning mathematics is a lot like learning to play an instrument, play a sport, or learn a language. You must practice everyday. The daily homework problems are intended to help you in your daily practice. But you should spend additional time each day working on the weekly homework, reading the text, and studying previous material. On average, you should spend 2 - 3 hours studying for each hour spent in class. In our case, that's 6-9 hours per week of homework and studying.

(2) *Participate in class.* Yes, you.

Asking and answering questions is a great way to stay engaged with the material and verify for yourself that you know what's going on. The more people that participate, the more fun class is. I, and the rest of the class, value your questions and your answers, right or wrong. In fact, giving a wrong answer to a questions is a great way to learn the right answer. Make an effort to connect each class's activities to previous classes. Try to predict where the material will be going in the future. Take good notes and listen. If you can't do both, make a deal with a buddy: you take notes and they listen one day and the next day you switch.

(3) *Read the textbook.*

The lectures and the textbook will often present slightly different views on the same material. The examples in the text will (for the most part) differ from the example presented in class. As you read the text, relate what the text does to what happened in class where is it the same, where is it different? If it differs, which approach do you like better– what are the pros and cons of each?

(4) *Form a study group.*

Introduce yourself to other people in the class and meet up outside of class to study and work on homework. Be sure that you don't copy answers, but learn from each other and then write the answers on your own. Compare lecture notes to be sure you copied everything correctly. Ask each other questions and explain material to each other. Calculus-After-Hours is for 100-level calculus students, but it makes the math department a happening place – it's a great place to meet up with your classmates.

(5) *Write your own problems.*

As you study for the exams, look back at all the examples done in class, in the text, or on homework. Try modifying them to make up your own problems. Try to solve your own problems what makes your problems easier or more difficult than the ones you've seen before? Feel free to show me what you've done.

(6) *Be curious.* We will be spending a lot of time, exploring the theory behind Vector Calculus. Try to be curious about why Calculus works. Ask "why" a lot. Feel free to ask about why we're learning something, but also ask about why the ideas came into existence and why the math works one way, but not another. Schedule time for curiousity. Block off 2 hours (most) every day for Calculus. Any time thats not spent working on homework or studying, use for playing with the ideas. Use a computer to sketch graphs of vector fields and curves – see if you can predict they way they'll look. Coming up with your own questions is a great way to learn to care about the underlying mechanics of Calculus. Computers can compute; humans ask creative questions.

(7) *Visit me in my office.*

I love working with students and I love to help you understand and appreciate the beautiful world of mathematics. Feel free to drop by, even when it's not my office hours. If I cant chat, Ill let you know. Ask crazy questions about the course. Ask questions about my research. Tell me about your past math experiences. Tell me about what subjects you love. Let me know when you have a concert or athletic event.

(8) *Spread the studying out over the semester.*

If you do math everyday, as suggested above, you wont have to cram for exams. Youll be able to sleep and, consequently, to think. You'll be happier and more relaxed. You'll have time to write papers for your other classes. You'll have time to appreciate the snow and the spring. You don't need to pull all-nighters.

(9) *Have an exam strategy.*

For the in-class exams, you will have only 50 minutes. Be prepared to do some problems very rapidly and be prepared to think about others. If an example was done in class or on homework, you should be able to repeat it very quickly on the exam. Know what you find difficult and what you find easy. Do the easy things first and then the difficult things. Write something for every problem. If you get stuck, tell me how you'd solve it if you could get unstuck. Figure out what the problem is testing and tell me what you know about that area. If the problem is too hard, rewrite it to make it easier. I love to give partial credit. Give me a reason to give you some. If you find yourself getting nervous: breathe deeply, remind yourself you've studied thoroughly, then figure out how to do the problem. Keep an eye on the time and don't spend too long on any one problem.