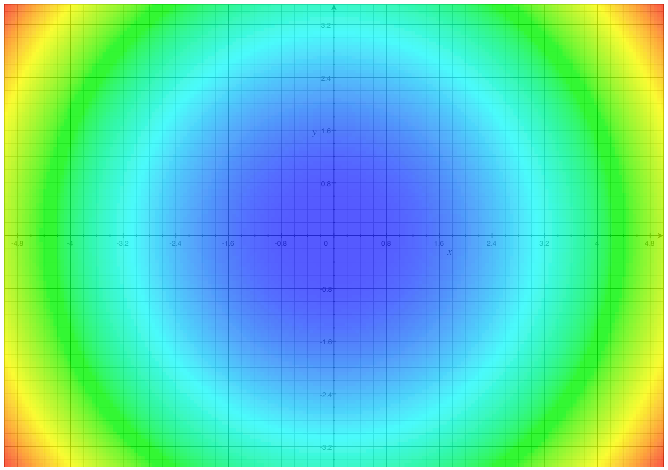
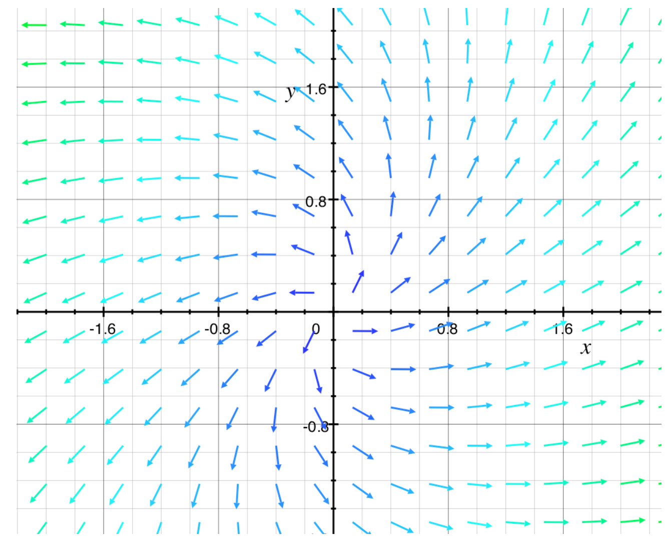
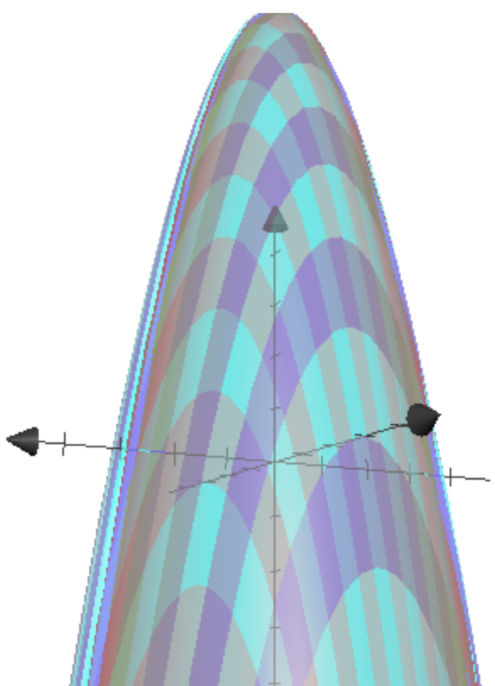
Vector Calculus

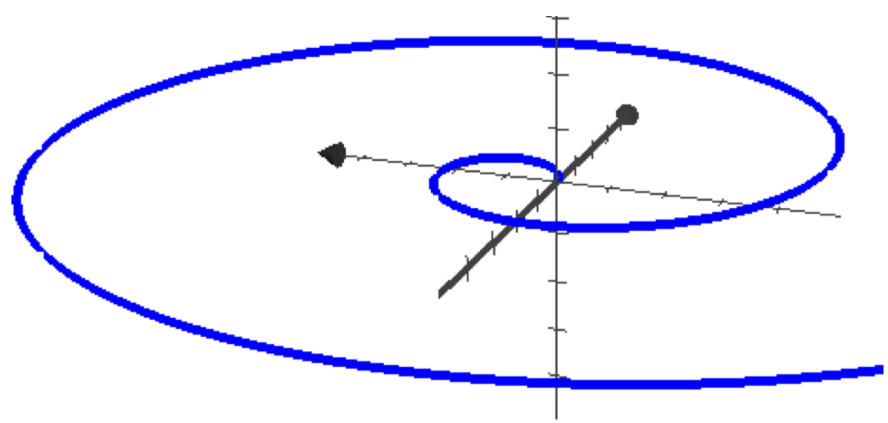
$$f : \mathbb{R}^n \to \mathbb{R}^m$$



Multiple Viewpoints







$$f : \mathbb{R}^n \to \mathbb{R}^m$$

Derivatives

$$f(x_1, x_2, \ldots, x_n) = \begin{pmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \ldots \\ f_m(\mathbf{x}) \end{pmatrix} \qquad Df(\mathbf{x}) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix}$$

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Linear approximation to f near $\mathbf{a}$.

$$\boxed{L(\mathbf{x}) = Df(\mathbf{a})(\mathbf{x} - \mathbf{a}) + f(\mathbf{a})}$$

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Linear approximation to f near $\mathbf{a}$.

$$L(\mathbf{x}) = Df(\mathbf{a})(\mathbf{x} - \mathbf{a}) + f(\mathbf{a})$$

Chain Rule

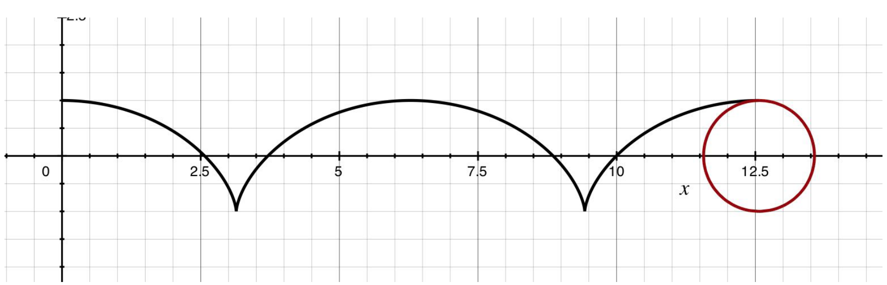
$$D(f \circ g)(\mathbf{a}) = Df(g(\mathbf{a})Dg(\mathbf{a})$$

Parameterizations

Curves

$$\mathbf{x} : \mathbb{R} \to \mathbb{R}^n$$

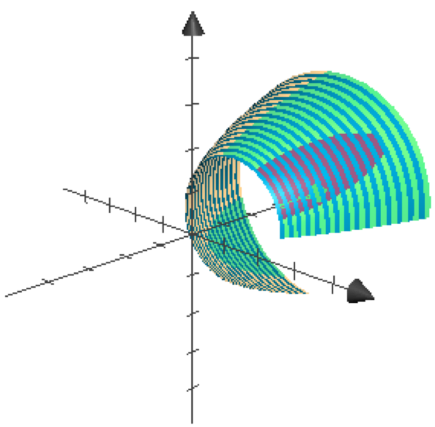
Circles
Graphs
Line Segments
Cycloids, etc.



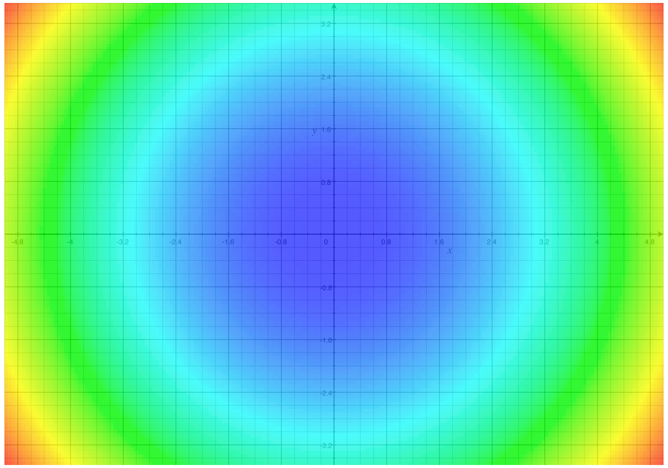
Surfaces

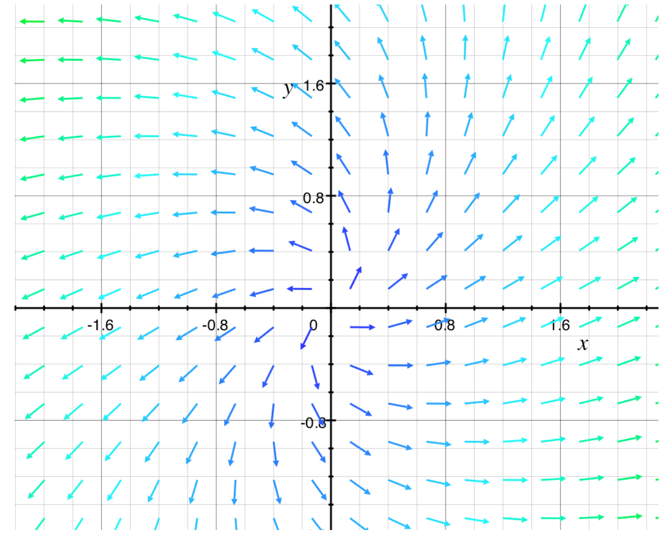
$$\mathbf{X} : \mathbb{R}^2 \to \mathbb{R}^n$$

Flat regions
Graphs
Surfaces of Revolution



Scalar Fields and Vector Fields





$\mathbb{R}^2$

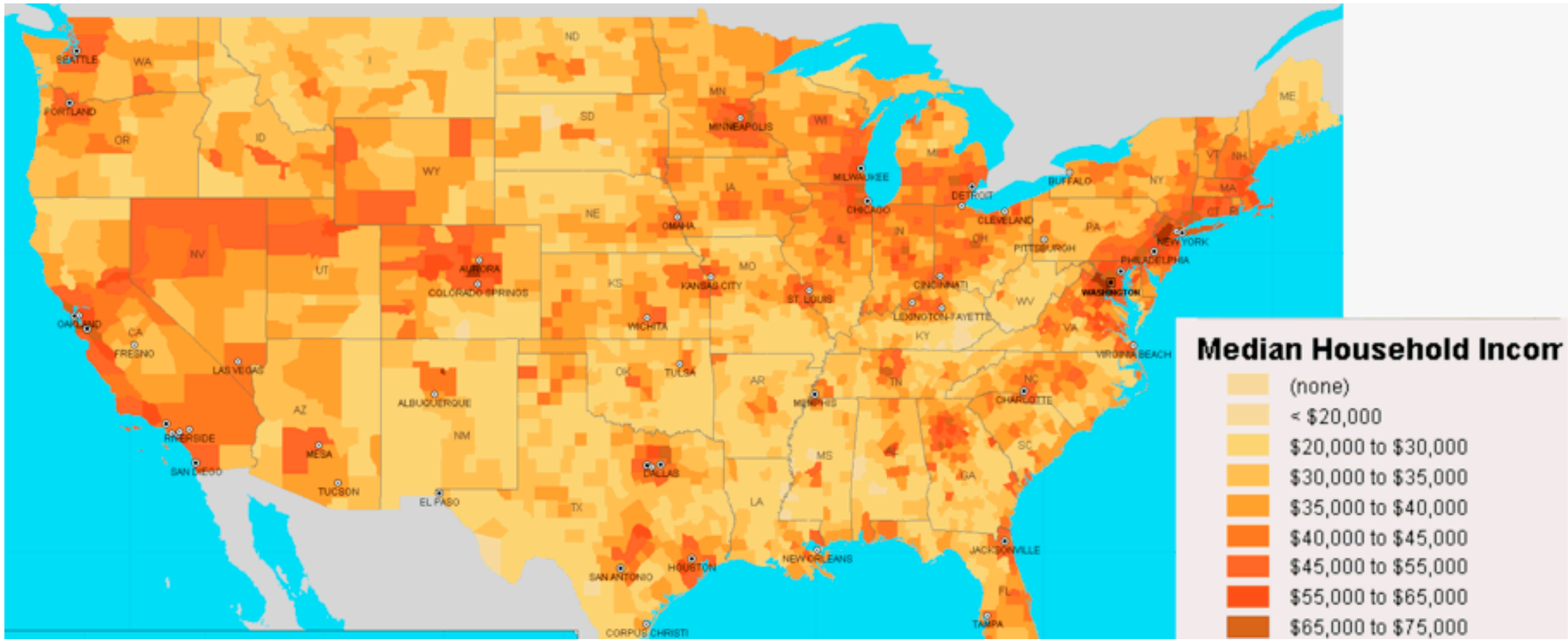
$$\boxed{\text{Scalar Fields}} \xrightarrow{\text{grad}} \boxed{\text{Vector Fields}} \xrightarrow{\text{scurl}} \boxed{\text{Scalar Fields}}$$

$\mathbb{R}^3$

$$\boxed{\text{Scalar Fields}} \xrightarrow{\text{grad}} \boxed{\text{Vector Fields}} \xrightarrow{\text{curl}} \boxed{\text{Vector Fields}} \xrightarrow{\text{div}} \boxed{\text{Scalar Fields}}$$

Scalar Fields and Vector Fields

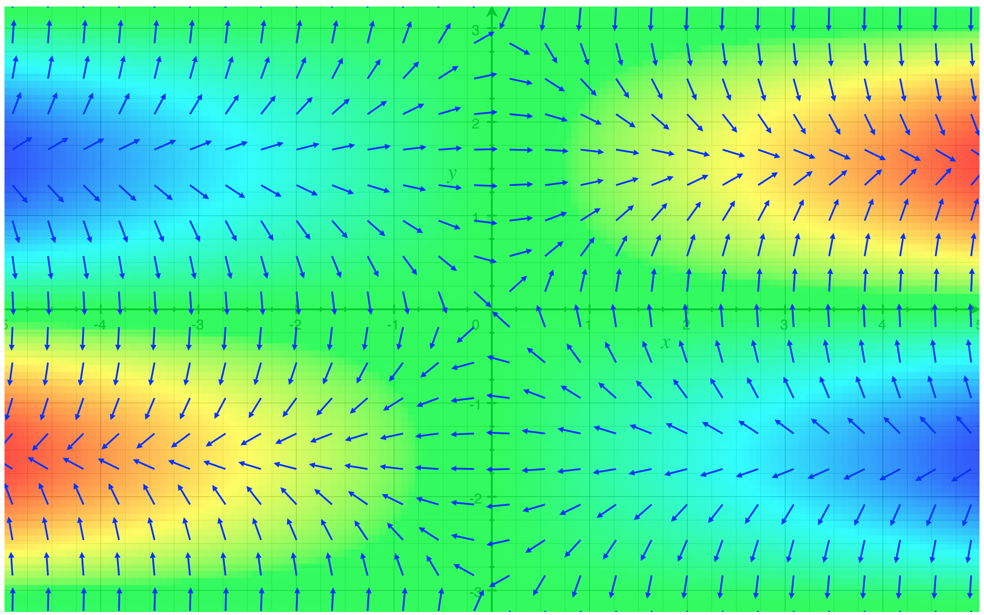
http://visualizingeconomics.com/2007/08/07/united-states-household-income-map/

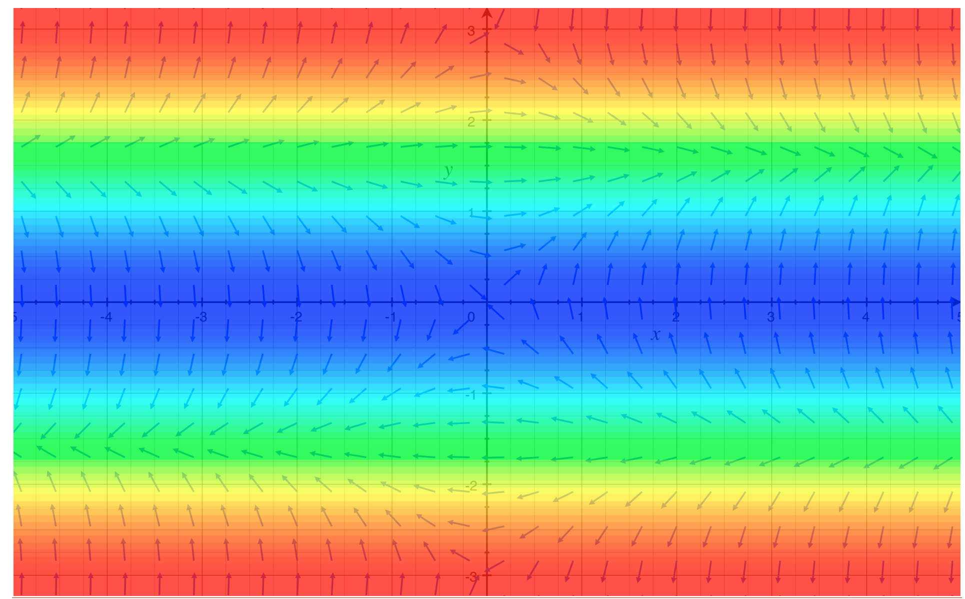


http://terrymarotta.wordpress.com/tag/snowstorm-no-tell-motels/

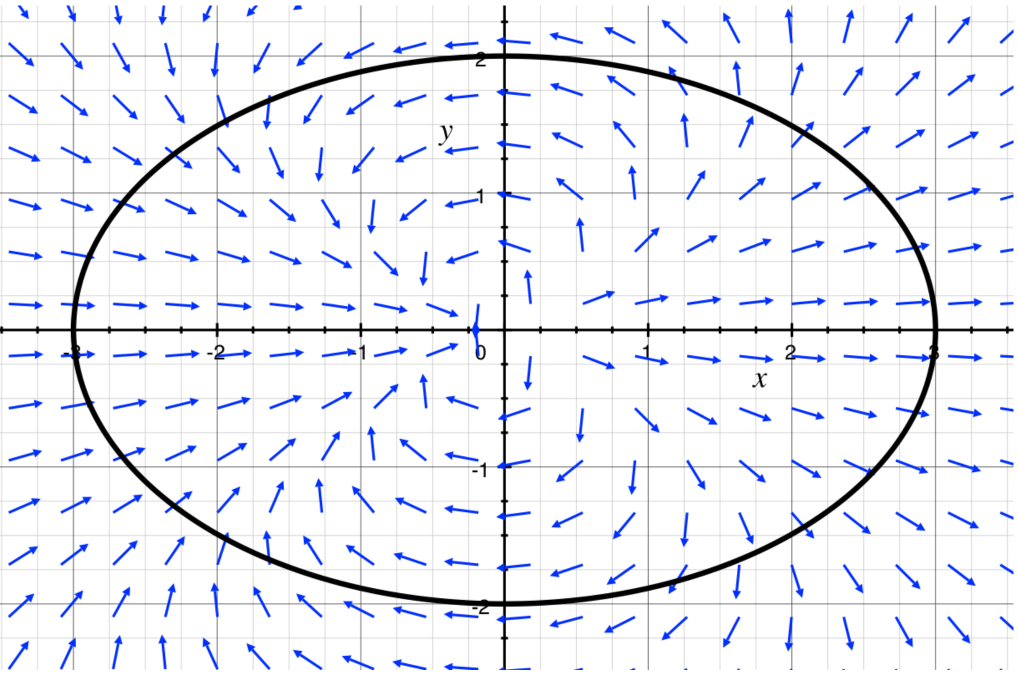
Scalar Fields and Vector Fields

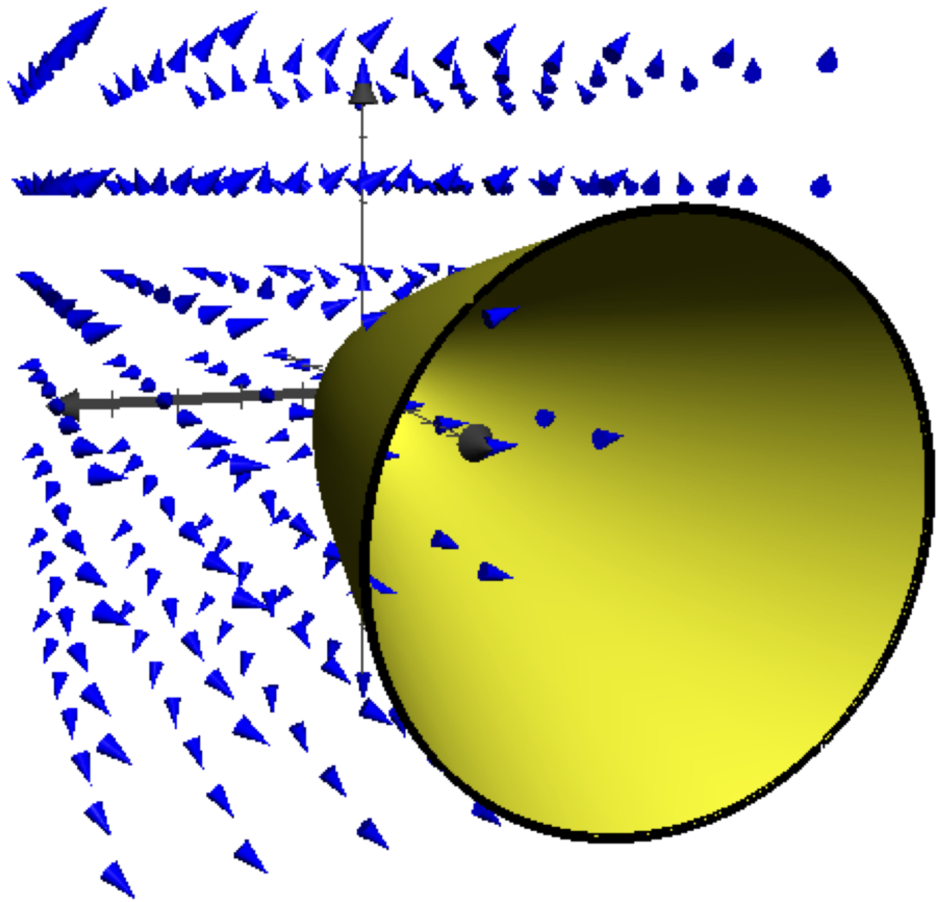
The Laplacian $\nabla^2 f = \operatorname{div} \nabla f$





Integrals





Integrals

FTC:

$$\int_{[a,b]} f'(t)\,dt = f(b) - f(a)$$

FTCCV:

$$\int_{\mathbf{x}} \nabla f \cdot d\mathbf{s} = f(\mathbf{x}(b)) - f(\mathbf{x}(a))$$

Green's:

$$\iint\limits_S \operatorname{scurl} \mathbf{F}\,dA = \int\limits_{\partial S} \mathbf{F} \cdot d\mathbf{s}$$

PDT:

$$\iint\limits_S \operatorname{div} \mathbf{F}\,dA = \int\limits_{\partial S} \mathbf{F} \cdot \mathbf{n}\,ds$$

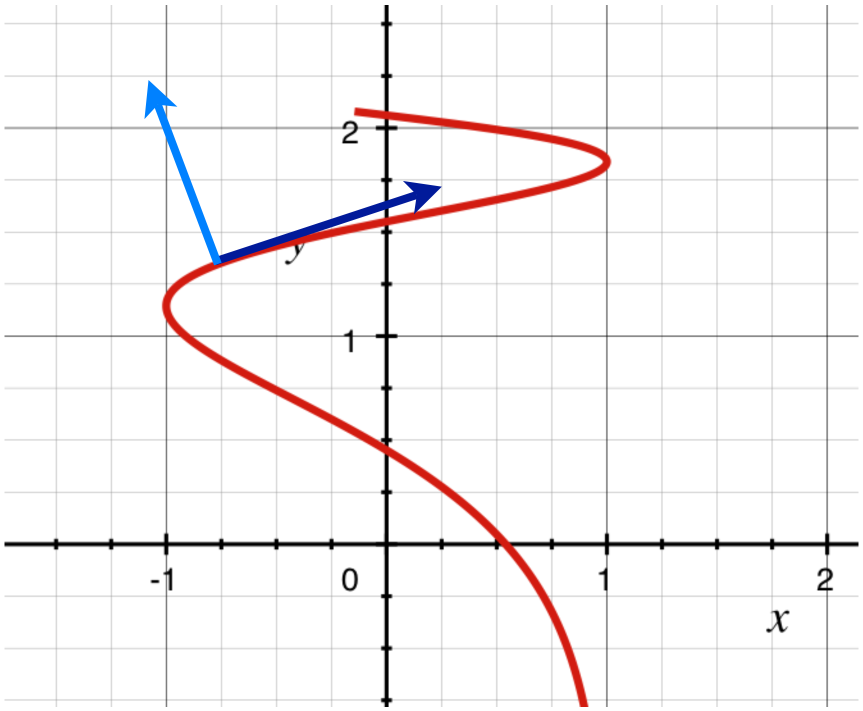
Stokes':

$$\iint\limits_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \int\limits_{\partial S} \mathbf{F} \cdot \ d\mathbf{s}$$

Divergence:

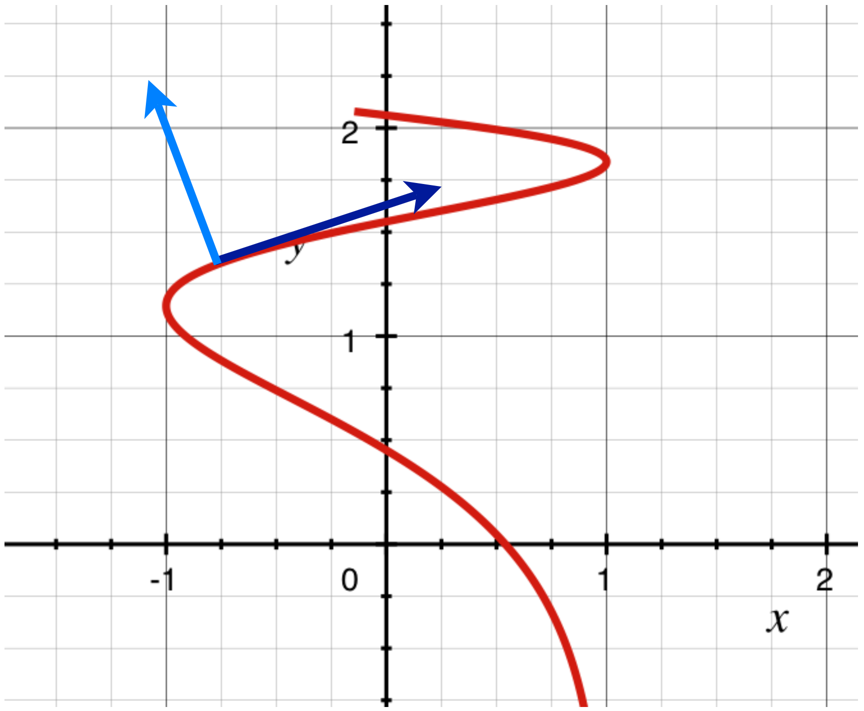
$$\iiint\limits_V \operatorname{div} \mathbf{F}\,dV = \iint\limits_{\partial V} \mathbf{F} \cdot \ d\mathbf{S}$$

Geometry...

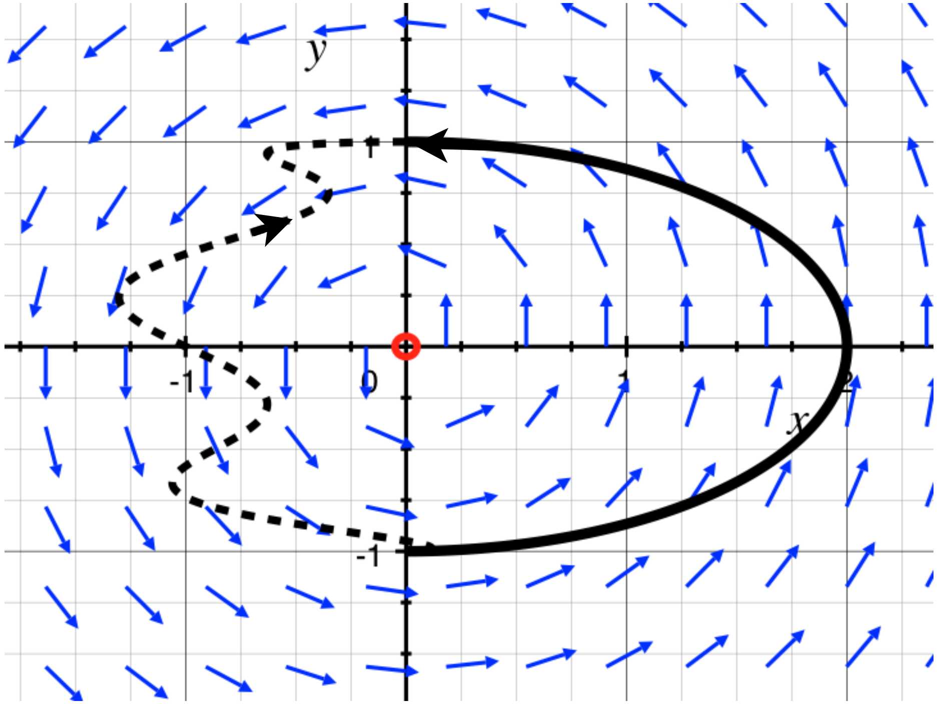


$$\mathbf{F}(x,y) = \frac{1}{x^2+y^2}\begin{pmatrix} -y \\ x \end{pmatrix}$$

Geometry...



and Topology



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