MA 262 Homework 5: Finding your path.

We have now entired the core content of the course: Vector Fields! Our goal for the next week is to develop an understanding of vector fields by studying paths moving through them.

Read Section 3.3 and 6.3 (except pg 393-394). As you read, pay attention to:

- The definition of gradient field and potential function
- How to find a flow line for a vector field.
- Properties of a conservative vector fieldd

Do these problems concerning work, flow lines and conservative vector fields.

pg 379: 16

pg 213: 20, 21, 23, 25

Note: in problem 23, an equipotential surface is the set of points where the potential function is constant. We prove in class that the vector field is always perpendicular to an equipotential surface.

Problem 5.A Recall that in class we defined the scalar curl of a vector field $\mathbf{F}$ on $\mathbb{R}^2$ by:

$$\operatorname{scurl}\mathbf{F}(\mathbf{a}) = \lim_{n\to\infty} \frac{1}{\operatorname{area}(C_n)} \int_{C_n} \mathbf{F}\cdot d\mathbf{s}$$

where (C_n) is a sequence of closed curves (preferably rectangles) converging to $\mathbf{a}$ and area(C_n) is the area enclosed by each of them. Use this definition in conjunction with the Fundamental Theorem of Calculus for Conservative Vector Fields to prove that if $\mathbf{F}$ is a conservative vector field then its scalar curl at each point is zero.

Problem 5.B(Bonus!) Suppose that $\mathbf{F}$ and $\mathbf{G}$ are two vector fields on $\mathbb{R}^2$ and that α and β are two real numbers. Use the integral formula for curl to prove that:

$$\operatorname{scurl}(\alpha\mathbf{F}+\beta\mathbf{G}) = \alpha\operatorname{scurl}(\mathbf{F})+\beta\operatorname{scurl}(\mathbf{G})$$

and

$$\operatorname{div}(\alpha\mathbf{F}+\beta\mathbf{G}) = \alpha\operatorname{div}(\mathbf{F})+\beta\operatorname{div}(\mathbf{G}).$$

(Those of you who have taken linear algebra may recognized that this problem shows that scalar curl and divergence are linear functions on the vector space consisting of vector fields on $\mathbb{R}^2$. Those of you who haven't taken linear algebra should look back at this problem when you have :))