MA 302: Project Topics

At the end of the semester, you will be giving an oral presentation and turning in a written document pertaining to material related to vector calculus. Below is a list of possible topics. Please rank your top 3 choices. I will place you into groups of 3 - 4 people and will assign you a project based on your rankings. If you would like any more information about these topics, please ask.

(a) _______ Improper Applications of Green's Theorem

Green's Theorem can be used to evaluate improper integrals that show up in 1-variable Calculus (for example, $\int_0^\infty (\sin x)/x\,dx$.) For this project, you will read one or two very short articles which show how this can be done.

(b) _______ Total Curvature of a Knot

At the age of 19, John Milnor (who was recently awarded the Abel Prize) proved a theorem now called the Fary-Milnor theorem (which had been proven slightly earlier by István Fáry). In this project you will define the notion of "total curvature" of a curve in $\mathbb{R}^n$ and discuss Milnor's proof that any knot with a total curvature of no more than 4π is unknotted.

(c) _______ Divergence and Gauss' theorem

In class we won't have a lot of time to discuss this generalization of the "planar divergence theorem", so in this project you'll develop these ideas as well as the related notion of "vector potential". The fundamental question to explore is: Suppose that $\mathbf{F}$ is a vector field in $\mathbb{R}^3$ with $\operatorname{div}\mathbf{F} = \mathbf{0}$. Is $\mathbf{F} = \operatorname{curl}\mathbf{G}$ for some vector field $\mathbf{G}$?

(d) _______ Complex Contour Integrals

The ideas used for studying line integrals in $\mathbb{R}^2$ can also be used for studying integration of complex-valued functions (leading to the beautiful subject of Complex Analysis). In this project, you will read about some of these connections and will present them in a form suitable to our class.

(e) _______ Maxwell's Equations

Maxwell's equations are fundamental to the theory of electromagnetism. In this project, you will explain what Maxwell's Equations say and will sketch their proof.

(Continued on reverse)

(f) _______ Alternative Geometries

If $\mathbf{x}: [a,b] \to \mathbb{R}^n$ is a curve, we learned that its length is given by $\int_a^b ||\mathbf{x}'(t)||\, dt$. In this project, you will explore different ways of calculating lengths and how these different ways lead to different kinds of geometry. You will explore different types of geometries on 2–dimensional surfaces and give some historical background.

(g) _______ (Co)homology theory

Noticing that the curl of the gradient is zero and that the divergence of curl is zero is the first step toward a comprehensive theory (known as cohomology) relating linear algebra to vector fields. Building off some of the additional problems from the homework, connections can also be made to the "algebra" of closed curves in a space (known as homology). In this project, you'll read some short notes, give an introduction to the basic ideas, and discuss why the theories are useful. (Co)homology theory is a vast subject, your project will be just the beginnings of an exploration. (In other words: don't panic if you look at the Wikipedia page!)

(h) _______ Your own idea

You may propose your own project which must be related to this course. Such a project may take the form of original research. For example, you may want to try to write down a rigorous proof of Green's Theorem (in particular, how does the limiting argument work?)