MA 302: Practice Exam 3 **Name:**________________________

These questions cover only the material since Exam 2. The first few problems are repeated from the previous practice exam.

(1) Find a parameterization of the surface formed by the graph of $z = x^2 - y^2$ with (x,y) in the triangle in the xy-plane formed by the x-axis, the y-axis, and the line $y = -x+1$.

(2) Is the surface in the previous problem a smooth surface? If no, at what points is it not smooth?

(3) Find a parameterization of the surface formed by rotating the curve $\begin{pmatrix} \cos t + 5 \\ 2\sin t \end{pmatrix}$ with $0 \leq t \leq 2\pi$ around the y-axis.

(4) Consider the surface

$$\mathbf{X}(s,t) = \begin{pmatrix} 2\sin 3t + t \\ \cos 2s \\ t^2 + s^2 \end{pmatrix}, \quad 0 \leq t \leq \pi/4, \quad 0 \leq s \leq \pi$$

Find the tangent and normal vectors to $\mathbf{X}$ at the point $(\pi/6, \pi/6)$. Is the surface smooth?

(5) Let S be the disc of radius 1 centered at $(1,0,0)$ in $\mathbb{R}^3$ which is parallel to the yz-plane. Orient S with normal vector pointing in the direction of the postive x-axis. Use the definition of surface integral to calculate the flux of $\mathbf{F}(x,y,z) = (-xy, yz, xz)$ through S.

(6) Use the same surface S and $\mathbf{F}$ as in the previous problem, but now use Stoke's theorem to calculate the flux of the curl from the previous problem.

(7) Give precise statements of Stokes' Theorem and the Divergence Theorem.

(8) State and prove Gauss' law for gravity.

(9) Use Gauss' Law for gravity and symmetry considerations to prove the shell theorem.

(10) Suppose that a vector field $\mathbf{F}$ defined on $\mathbb{R}^3 - \{\mathbf{0}\}$ has a flux of 21 through a sphere of radius 2 (oriented outward). If the divergence of

F is a constant -1, what is the flux of **F** through a sphere of radius 4 (oriented outward)?

(11) Suppose that **F** is a C^1 vector field that is everywhere tangent to the unit sphere in $\mathbb{R}^3$. Explain why the flux of **F** through the sphere must be zero. If **F** is also C^1 everywhere inside the sphere, what can you conclude about the divergence of **F** inside the sphere?

(12) Suppose that **F** is a C^1 vector field and that S is a compact surface without boundary. If the circulation of **F** around S is non-zero, what can you conclude about S?

(13) Suppose that two surfaces S_1 and S_2 have the same oriented boundary and that they are disjoint except along their boundaries. Suppose that **F** is a C^1 vector field defined on the region bounded by the union of S_1 and S_2. Explain why the circulation of **F** is the same on S_1 and S_2. If the vector field is incompressible, explain why the flux through S_1 is the same as the flux through S_2.