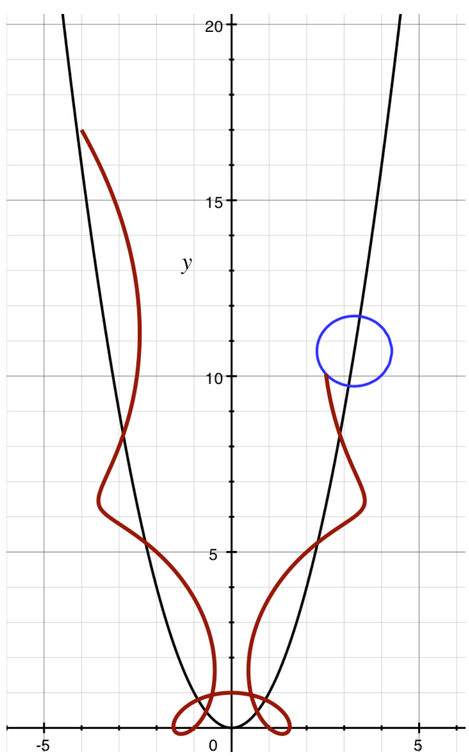
MA 302: HW 2 additional problems

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem 2.A: Suppose that a circle of radius 1 is rolling down a hill such that the center of the circle is always on the graph of the parabola $y = x^2$. The circle rolls in such that the center of the circle is at the point (t, t^2) at time t and it completes 1 clockwise rotation every 2 seconds. At time $t = 0$, the center of the circle is at the point $(0, 0)$. Let P be the point on the circle directly above the center of the circle at time $t = 0$. Find the parameterization of the path $\mathbf{x}(t)$ taken by the point P as the circle rolls down the parabola. For extra credit use Grapher to make an animation of the circle rolling down the parabola and the path taken by P. You should email the grapher file to me. An example of a still from your animation might be:



Problem 2.B: *The ride*

One of the most famous rides at Disney Land/World is the Mad Tea Party. Consider a version of this ride which consists of a large disc A that rotates counter-clockwise (viewed from above). Inside that disc and tangent to it are four smaller discs all (nearly) tangent to each other. These smaller discs rotate counter-clockwise (viewed from above, not accounting for the motion of A). Let B be one of these smaller discs. Inside B and tangent to it are four smaller circles (the tea cups), all (nearly) tangent to each other. These smaller circles spin at a variable rate determined by the people inside the teacup. Let C be one of them. Let P be a point on C (representing the position of someone on the ride). See the figure[1].

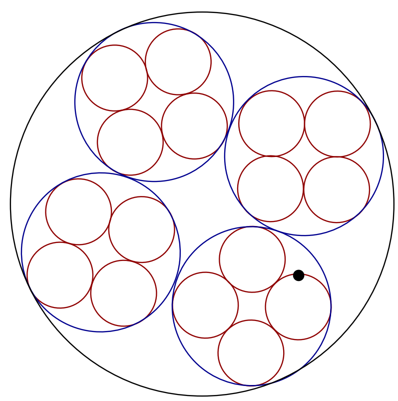


FIGURE 1. The point P is marked with a black dot

Measurements

Suppose that units are chosen so that the radius of A is 1. Then the radius of B is $r_b = \frac{1}{1+\sqrt{2}}$ and the radius of C is $r_c = \frac{1}{(1+\sqrt{2})^2}$.

Suppose that A rotates at 1 revolution per second and that B rotates at 2 revolutions per second. The circle C rotates at $r(t)$ rotations per second, where if $r(t) < 0$, then C is rotating clockwise.

The centers

Choose coordinates so that the center of A is at the origin. Let $\mathbf{b}(t)$ denote the center of B at time t and let $\mathbf{c}(t)$ denote the center of C at time t. Let $\mathbf{P}(t)$ denote the position of P at time t. We assume that at time $t = 0$, the circles A, B, and C are all tangent to each other at the point $(1,0) = \mathbf{P}(0)$.

[1] In the actual ride, there are only 3 discs of Type B and they are not tangent and there are 5 teacups (not tangent) inside each of those discs.

The problems

(1) (Extra-credit) Use elementary geometry to determine the radii of B and C given in the measurements section above.

(2) Find the coordinates of $\mathbf{b}(t)$. (Hint: The center of B always lies on a circle that is concentric with A and it moves at the same angular rate (and in the same direction) as A.)

(3) Find the coordinates of $\mathbf{c}(t)$. (Hint: The center of C always lies on a circle that is concentric with B and it moves at the same angular rate (and in the same direction) as B.)

(4) In tangent space coordinates based at $\mathbf{c}(t)$, find the coordinates of $\mathbf{P}(t)$. (Your answer will depend on $r(t)$, but not on any of the previous problems.)

(5) Find the coordinates of $\mathbf{P}(t)$ in the usual coordinate system. Your answer will, of course, depend on $r(t)$.

(6) It turns out that if $r(t) = r$ is a constant, at time $t = 0$, we have:

$$||\mathbf{P}'(0)||^2 = \frac{4\pi^2(r+3\sqrt{2}+2)^2}{(1+\sqrt{2})^4}$$

Suppose that you are riding on the teacups with a 3-year old who is scared to travel too fast. How fast (in revolutions per second) and in what direction should you make the teacup rotate so as to minimize your speed at time $t = 0$? (You may restrict yourself to the situation where $r(t)$ is a constant independent of t.) Be sure to explain how you got your answer.