MA 302: Final Exam Review **Name:**______________________________

(1) Derivatives:

  (a) Understand and be able to calculate the derivative as a matrix
  (b) Understand the definition of C^1 and of differentiable
  (c) Be able to find the equation for the affine approximation to a function at a point
  (d) Know and be able to use the chain rule

(2) Parameterized Curves

  (a) Know parameterizations for common curves (circles, straight lines, graphs of functions)
  (b) Understand and be able to use tangent space coordinates to find the parameterizations of complicated curves (epicycles, cycloids, etc.)
  (c) Understand what the derivative of a parameterized curve measures
  (d) Be able to reparameterize a curve with an orientation preserving or reversing change of coordinates function.
  (e) Understand the difference between intrinsic and extrinsic properties of curves
  (f) Be able to write down an integral representing the length of a parameterized curve.

(3) The geometry of parameterized curves

  (a) Be able (in practice and principle) to reparameterize a curve by arclength.
  (b) Be able to prove that the unit tangent vector **T** is intrinsic to oriented curves
  (c) Be able to calculate the moving frame **T**, **N**, and **B** (although **B** won't be on the exam.)
  (d) Be able to calculate curvature $\kappa(t)$.
  (e) Understand the idea of tangential and normal components to acceleration.
  (f) Be able to prove that in a 2-body system consisting of a planet and a sun, the planet's orbit will lie in a plane.

(4) Line Integrals

(a) Know that if f is a scalar field and if $\mathbf{x}\colon [a,b] \to \mathbb{R}^n$ is a path then
$$\int_{\mathbf{x}} f\,ds = \int_a^b f(\mathbf{x}(t))||\mathbf{x}'(t)||\,dt.$$
(b) Know that if $\mathbf{F}$ is a vector field and if $\mathbf{x}\colon [a,b] \to \mathbb{R}^n$ is a path then
$$\int_{\mathbf{x}} \mathbf{F}(\mathbf{x}(t)) \cdot \mathbf{x}'(t)\,dt.$$
(c) Understand what the path integral of a vector field measures (work, circulation, etc.) and why.

(5) Vector Fields

(a) Be able to draw a picture of a given vector field
(b) Know the concept of "flow line" and be able to work simple examples
(c) Understand what curl measures and be able to calculate it.

(6) Green's Theorem

(a) Know the precise statement of Green's Theorem
(b) Be able to calculate the integrals appearing on both sides of the equality in Green's theorem (that is: "verify" Green's theorem for particular examples.)
(c) Be able to use the integral on one side of Green's theorem to calculate the integral on the other side (that is: "use" Green's theorem for particular examples.)
(d) Use Green's theorem to find areas enclosed by curves
(e) Use Green's theorem to relate the line integral of a vector field around one curve to the line integral of the vector field around a different curve
(f) Be able to explain why Green's theorem is true by identifying the important features of the proof and how they fit together.

(7) Conservative Vector Fields

(a) Know what a conservative/gradient field is and be able to find potential functions for simple examples
(b) Know the basic idea for why conservative vector fields don't have closed up flow lines
(c) Be able to prove that the line integral of a conservative field over an equipotential curve is 0.
(d) Be able to prove that conservative vector fields have path independent line integrals.

(e) Be able to construct the integral form of a potential function for a vector field with path independent line integrals.
(f) Be able to apply the theory of conservative vector fields to calculate the work done in moving a particle through the electric field generated by a charged wire.
(g) Be able to prove that a vector field is conservative if and only if its integral around any closed curve is 0.
(h) Be able to prove that on a simply connected domain if a vector field has zero curl then it is conservative.
(i) Use curl to determine whether or not a vector field is conservative
(j) State Poincaré's theorem
(k) Be able to give an example of a vector field having curl zero on its domain which is not a conservative vector field. Understand the relationship between this example and Poincaré's theorem

(8) Planar Divergence Theorem

(a) Know and be able to explain the statement of the planar divergence theorem
(b) Be able to both "verify" and "use" the planar divergence theorem
(c) Understand and be able to explain the relationship between the Green's theorem and the planar divergence theorem.

(9) Curves, Vector Fields, and 2-dimensional regions

(a) Understand ways in which the holes in a 2-dimensional region can affect the behaviour of vector fields having curl zero.

(10) Surface Integrals and Stokes' theorem

(a) Know the definitions of "surface" in both the topological and calculus senses
(b) Be able to find write down equations for standard parameterizations of surfaces
(c) Be able to match a parameterization with the image of a surface.
(d) Know what it means to "orient" a surface and know examples of non-orientable surfaces
(e) Know and be able to use the definition of surface integral of a scalar field and vector field
(f) Understand how the change of variables theorem can be used to show that a surface integral of a vector field is intrinsice to an (oriented) surface.

(g) Know the statement of Stokes' theorem and the Divergence Theorem
(h) Be able to use Stokes' theorem to calculate the circulation of a vector field over a surface
(i) Be able to use Stokes' theorem to calculate the line integral of a vector field over a curve bounding a surface
(j) Be able to use the divergence theorem to calculate the flux of a vector field through a closed surface
(k) Be able to use the divergence theorem to relate the flux of a vector field through one surface to the flux through a different surface having the same oriented boundary

(11) Know the answers to the project questions distributed in class (and posted on the website).