

(1) Derivatives:

- (a) Understand and be able to calculate the derivative as a matrix
- (b) Understand the definition of  $C^1$  and of differentiable
- (c) Be able to find the equation for the affine approximation to a function at a point
- (d) Know and be able to use the chain rule

(2) Parameterized Curves

- (a) Know parameterizations for common curves (circles, straight lines, graphs of functions)
- (b) Understand and be able to use tangent space coordinates to find the parameterizations of complicated curves (epicycles, cycloids, etc.)
- (c) Understand what the derivative of a parameterized curve measures
- (d) Be able to reparameterize a curve with an orientation preserving or reversing change of coordinates function.
- (e) Understand the difference between intrinsic and extrinsic properties of curves
- (f) Be able to write down an integral representing the length of a parameterized curve.

(3) The geometry of parameterized curves

- (a) Be able (in practice and principle) to reparameterize a curve by arclength.
- (b) Be able to prove that the unit tangent vector  $\mathbf{T}$  is intrinsic to oriented curves
- (c) Be able to calculate the moving frame  $\mathbf{T}$ ,  $\mathbf{N}$ , and  $\mathbf{B}$  (although  $\mathbf{B}$  won't be on the exam.)
- (d) Be able to calculate curvature  $\kappa(t)$ .
- (e) Understand the idea of tangential and normal components to acceleration.
- (f) Be able to prove that in a 2-body system consisting of a planet and a sun, the planet's orbit will lie in a plane.

## (4) Line Integrals

- (a) Know that if  $f$  is a scalar field and if  $\mathbf{x}: [a, b] \rightarrow \mathbb{R}^n$  is a path then

$$\int_{\mathbf{x}} f ds = \int_a^b f(\mathbf{x}(t)) \|\mathbf{x}'(t)\| dt.$$

- (b) Know that if  $\mathbf{F}$  is a vector field and if  $\mathbf{x}: [a, b] \rightarrow \mathbb{R}^n$  is a path then

$$\int_{\mathbf{x}} \mathbf{F}(\mathbf{x}(t)) \cdot \mathbf{x}'(t) dt.$$

- (c) Understand what the path integral of a vector field measures (work, circulation, etc.) and why.

## (5) Vector Fields

- (a) Be able to draw a picture of a given vector field
- (b) Know the concept of “flow line” and be able to work simple examples
- (c) Understand what curl measures and be able to calculate it.
- (d) Know what a conservative/gradient field is and be able to find potential functions for simple examples
- (e) Know how the basic idea for why conservative vector fields don't have closed up flow lines
- (f) Be able to prove that the line integral of a conservative field over an equipotential curve is 0.
- (g) Be able to prove that conservative vector fields have path independent line integrals.