MA 121: Final Exam **Name:**____________________________________

Instructions: The exam is closed book, closed notes, although you may use a note sheet as in the previous exam. A calculator is allowed, but you must show all of your work. **Your work is your answer.** If you have any questions, please ask immediately! Good luck.

Problem	Score	Possible
1		25
2		20
3		40
4		15
5		20
6		20
7		20
8		20
9		20
10		10 (bonus!)
Total		200

Problem #1:
Calculate the following derivatives:

(i)

$$\begin{aligned} f(x) &= \sin(x)\ln(x) \\ \tfrac{d}{dx} f(x) &= \end{aligned}$$

(ii)

$$\begin{aligned} \cot(\theta) &= \tfrac{\cos(\theta)}{\sin(\theta)} \\ \tfrac{d}{d\theta} \cot(\theta) &= \end{aligned}$$

(iii)

$$\begin{aligned} r(t) &= e^{5t^2-3} \\ \tfrac{d}{dt} r(t) &= \end{aligned}$$

Calculate the following derivatives:

(iv)

$$\begin{aligned} q(z) &= \arctan(\ln(z^5+3)) \\ \tfrac{d}{dz}\, q(z) &= \end{aligned}$$

(v)

$$\begin{aligned} b(x) &= \textstyle\int_2^x \ln(t^3-t)\,dt \\ \tfrac{d}{dx}\, b(x) &= \end{aligned}$$

Problem #2:
Find the following antiderivatives or definite integrals:

(i)

$$\int_0^{\pi/4} \sin(t)\, dt$$

(ii)

$$\int 3e^{5x}\, dx$$

(iii)

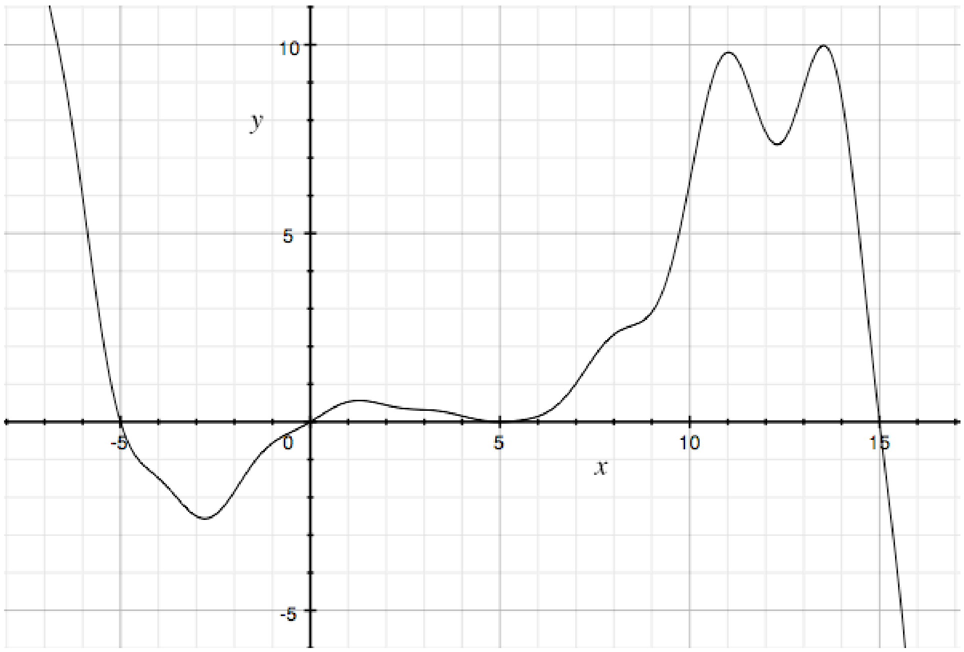
$$\int_0^{\pi/2} \sin(\cos(t))\sin(t)\, dt$$

Find the following antiderivatives or definite integrals:

(iv)

$$\int_0^4 \frac{1}{2+2\sqrt{t}}\,dt$$

Problem #3:
The graph of a function $f(x)$ is drawn. Let $F(x)$ be an antiderivative of $f(x)$. Answer the following questions about $f'(x)$, and $F(x)$. Consider only x values such that $-6 \leq x \leq 16$.



(i) For what values of x is $f'(x) = 0$? If you are unsure about some points list them with a question mark.

(ii) On the interval $[10, 11]$ is $f'(x)$ positive or negative? Give a reason for your answer.

(iii) On the interval $[10, 11]$ is $f'(x)$ increasing or decreasing? Give a reason for your answer.

(iv) List all x values such that $F(x)$ has a local maximum.

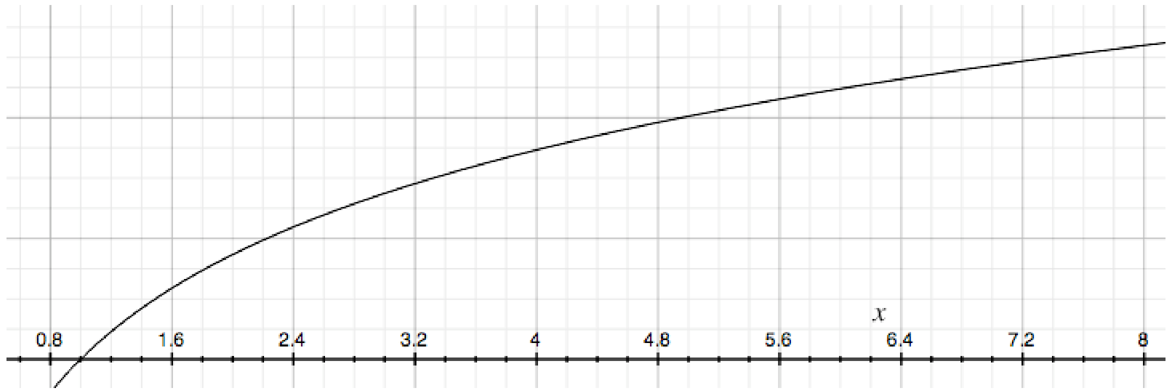
(v) List all x values such that $F(x)$ has a local minimum.

(vi) List all x values such that $F(x)$ has a critical point which is neither a local maximum or a local minimum.

(vii) For what x values is $F(x)$ decreasing? (You should list one or more intervals.)

(viii) For what x values is $F(x)$ concave down? (You should list one or more intervals.)

Problem #4:
Let $f(x) = \ln(x)$. Here is the graph:



(i) Find the equation of the line tangent to the graph of $f(x)$ at the point $(e^2, 2)$.

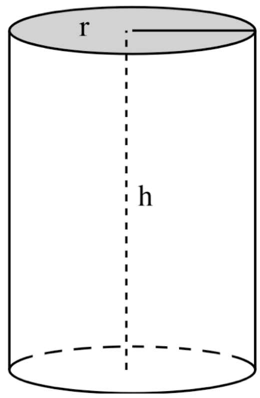
(ii) Use your answer from (i) to approximate $\ln(7)$. (Hint: $e^2 \approx 7.4$.)

(iii) Is your answer in (ii) too big or too small? Use the graph of $y = \ln x$ (shown above) to explain your answer. (No credit will be given for explanations that do not refer to the graph in some way.)

Problem #5:

A cylindrical box is going to be built to house the arm of St. Philip. The box must have a volume of 400 in^3. What radius should the cylindrical box have in order to minimize the surface area of the box (including the bottom and top)?

The formula for the volume of a cylinder is $V = \pi r^2 h$ and the formula for the total surface area is $A = 2\pi rh + 2\pi r^2$. Here r is the radius of the base and top of the cylinder and h is the height of the cylinder. (Hint: Your answer will look unpleasant and will involve a cube root.)



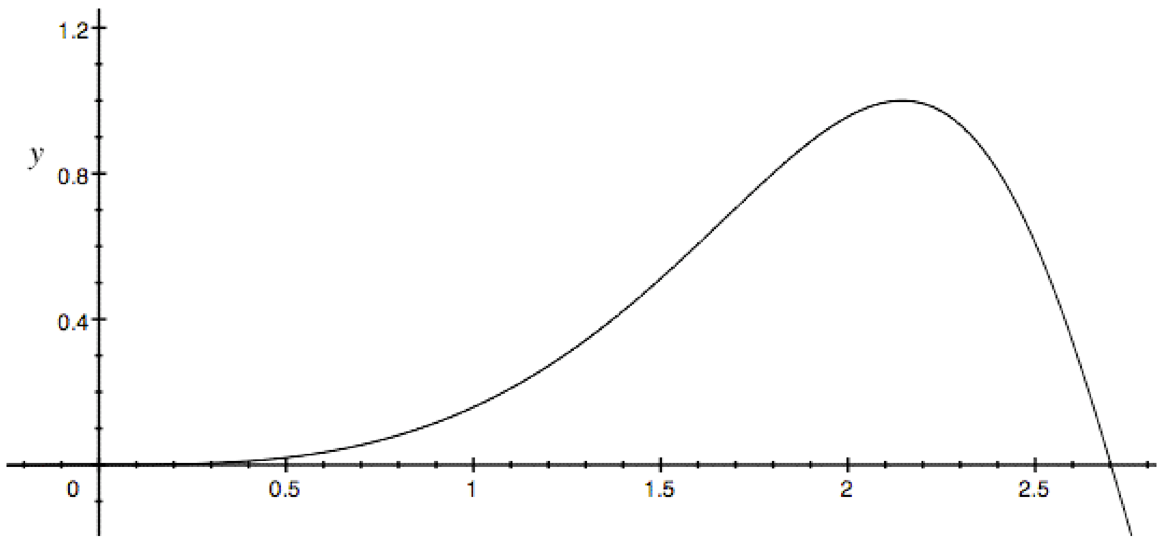
Problem #6:

A certain trash compactor is a rectangular box which is 40 meters deep from north to south. Fetid water is being added to the box at the rate of 100 m^3 per minute. The east and west walls of the compactor are approaching each other. The distance between them is decreasing at a rate of 10 meters per minute. How fast is the height of the water rising when the water is 5/4 meter deep and the east/west walls are 3 meters apart?

Problem #7:

Approximate the following integral using $n = 4$ boxes of equal width. The graph of the function is shown below. You do not need to perform the actual computations – simply write down what computations need to be done.

$$\int_0^2 \sin(x^3/\pi)\, dx$$



Problem #8:
In class (at various times) we discussed the following three important theorems:

Extreme Value Theorem (EVT)

Mean Value Theorem (MVT)

Intermediate Value Theorem (IVT)

(i) Two of the three theorems apply (primarily) to continuous functions. One of them applies to differentiable functions. Which one applies to differentiable functions?

(ii) Which theorem is used to prove that if $F'(x) = 0$ then there is a constant C so that $F(x) = C$?

(iii) Which of them did we use to help prove the first version of the fundamental theorem of calculus?

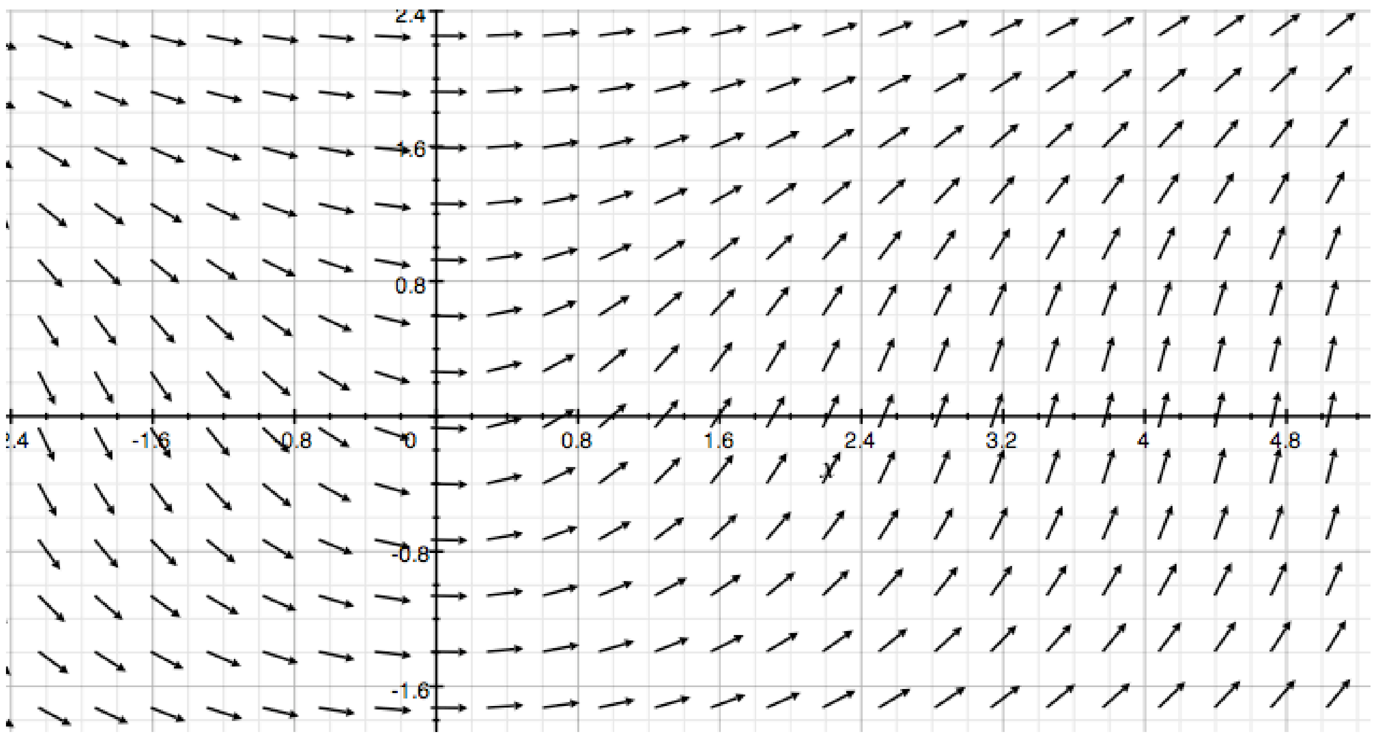
(iv) Which of them can be used to show that the function

$$f(x) = x^{1001} - x^2 + 89$$

crosses the x–axis?

Problem #9:
Here is a slope field for the differential equation

$$y' = \frac{x}{y^2+1}$$



(i) On the slope field, sketch the graph of the solution to the differential equation which passes through the point $(0, -1)$.

(ii) Solve the differential equation

$$y' = \frac{x}{y^2+1}$$

(You do not need to solve for y in your solution.)

Problem #10:

(Bonus!) For a function $f(x)$ which is always positive. The amount of area between the graph of the function and the x–axis on the interval $[a, \infty)$ is represented by

$$\lim_{t \to \infty} \int_a^t f(x)\, dx.$$

(i) Is the area between the graph of $y = 1/x$ and the x–axis on the interval $[1, \infty)$ finite or infinite?

(ii) Is the area between the graph of $y = 1/x^2$ and the x–axis on the interval $[1, \infty)$ finite or infinite?