MA 121: Practice Exam 2 **Name:**________________________________

Here are some practice problems. Some of them are more difficult than the problems that will appear on the exam. There are certainly more problems here than will appear on the exam.

(1) Write down the right Riemann sum with $n = 5$ rectangles for the function $f(x) = \frac{1}{\sqrt{x}}$ on the interval $[1,5]$.

(2) Use a left Riemann sum with $n = 3$ rectangles to approximate the area between the graph of $f(x) = \sqrt[3]{x}$ on the interval $[0,1]$.

(3) Use the definition of the definite integral as the limit of Riemann sums to calculate $\int_0^2 x^2\,dx$. I suggest you use right Riemann sums. You will need to know that $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$.

(4) Suppose that f is a continuous function. Find $\lim_{h\to 0} \frac{1}{h}\int_x^{x+h} f(t)\,dt$.

(5) Suppose that f is a continuous function. State and prove the first version of the fundamental theorem of Calculus.

(6) Find the derivative with respect to x of the following functions. For give a rationale for your answer:

(a) $\int_0^x \sin(t^2)\,dt$
(b) $\int_x^0 \sin(t^2)\,dt$
(c) $\int_{-x}^x \sin(t^2)\,dt$.

(7) Prove that if f is a differentiable function such that $f'(x) = 0$ for all x, then f is a constant function.

(8) Prove that if $F'(x) = G'(x)$ for all x, then there is a constant C such that $F(x) = G(x) = C$.

(9) Use only the first version of the fundamental theorem of Calculus and the previous problem to find $\int_0^2 t^2\,dt$.

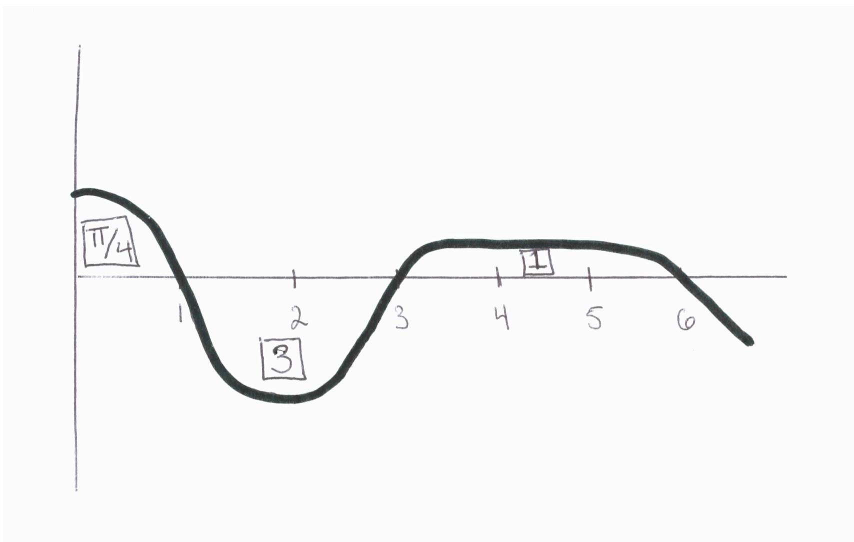
(10) Recalling that $\ln(x) = \int_1^x \frac{1}{t}\,dt$, explain why $\frac{d}{dt}\ln(x) = \frac{1}{x}$. What is the equation of the tangent line to the graph of $y = \ln(x)$ at the point $(e,1)$?

(11) Recall that e^x is defined to be the inverse function of $\ln(x)$. Use this fact and the previous problem to prove that the derivative of $g(x) = e^x$ at $x = 1$ is $g'(1) = e$.

(12) A differentiable one-to-one function $f(x)$ has the properties that $f(1) = 7$ and $f'(1) = 3$. Carefully explain, using pictures, why the derivative of $f^{-1}(x)$ at $x = 7$ is $1/3$.

(13) Prove that $\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$.

(14) Drawn below is the graph of a function $f(x)$, with the area between the graph and the x axis marked for various intervals. Suppose that $F(x)$ is an anti-derivative of $f(x)$ with $F(0) = 6$. Find $F(6)$.



(15) Let $F(x) = \int_0^x t(t+2)(t-1)\,dt$. Find and classify all critical points of $F(x)$.

(16) Using the fact that $\frac{d}{dt}\sin t = \cos t$, prove that $\frac{d}{dt}\cos t = -\sin t$.

(17) Find derivatives (with respect to t) of the following functions:

(a) $f(t) = \sin t + \cos t$
(b) $g(t) = e^t \sin t$
(c) $h(t) = \sqrt{e^t - 1}$
(d) $k(t) = \sqrt{e^{t^2} - 1}$
(e) $m(t) = \frac{\sqrt{t}}{\sqrt{t^2-1}}$
(f) $n(t) = \sin(\ln t)$

(18) Find $\frac{dy}{dx}$ for the curve $x^3 - y^2 = 1$ at the point $(2, \sqrt{7})$.

(19) Find the following antiderivatives:

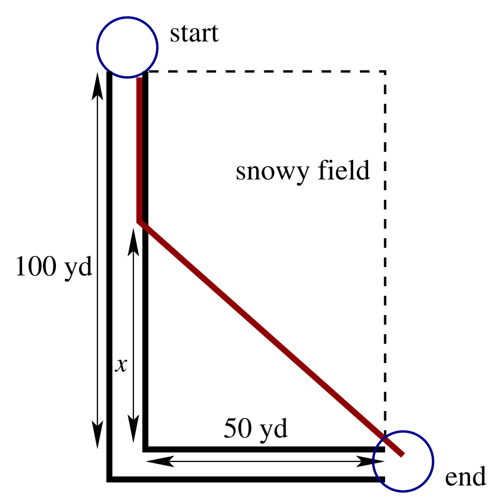
(a) $\int t^2 - \sqrt{t} + \frac{1}{t}\,dt$.
(b) $\int e^{2t}\sin(e^{2t})\,dt$
(c) $\int \sin^2 t\cos t\,dt$
(d) $\int \frac{\arcsin t}{\sqrt{1-t^2}}\,dt$

(20) Find the following definite integrals:

(a) $\int_1^3 x^2 - \frac{1}{\sqrt[3]{x}}\,dx$
(b) $\int_0^{\pi/4}\sin(2t)\,dt$
(c) $\int_0^8 te^{t^2}\,dt$
(d) $\int_0^3 t\sin(t)\,dt$

(21) Prove that every function $f(x)$ having the property that $f'(x)$ is proportional to $f(x)$ is of the form $f(x) = Ae^{kx}$ for some constants A and k. Suppose that a population of bacteria doubles every 1/2 hour. Explain the relevance of the the equation $f'(x) = kf(x)$ to determining the population of the bacteria after 75 minutes.

(22) In walking from Colby to my house I need to walk by the Mount Merici school. See the diagram below. I can cut straight across the field through deep snow, to my house, or I can walk along the road with no snow. Of course, I can also walk along the road for some distance, and then cut through the field.



If I walk at 90 yards per minute with no snow and at 30 yards per minute through deep snow, what path gets me to my house in the shortest amount of time? That is, how far should I walk along the road, before cutting through the snow? Solve this problem in several steps:

(a) Suppose that I choose to start cutting through the field, x yards before the corner. How much time do I spend walking on the road?
(b) Suppose that I choose to start cutting through the field, x yards before the corner. What distance do I travel through the field? How much time do I spend walking through the field?
(c) Let $s(x)$ be the total time spent getting to my house from the start of Mt. Merici's driveway, where I choose to start cutting through the field x yards before the corner. Find a formula for $s(x)$.
(d) Find the critical points of $s(x)$. Is one of these the global minimum? If not what is?