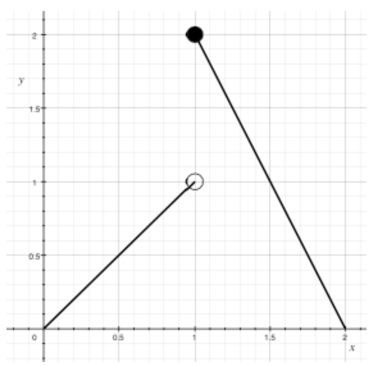
MA 121: Practice Exam 1 **Name:**____________________________________

Here are some practice problems. Some of them are more difficult than the problems that will appear on the exam. There are certainly more problems here than will appear on the exam.

(1) Here is the graph of a function $f(x)$. Use both words and pictures to give an $\varepsilon - \delta$ argument that $\lim_{x\to 1} f(x) \neq 1.5$.



(2) Use the limit definition of the derivative to prove that the derivative of $f(x) = x^2$ is $f'(x) = 2x$.

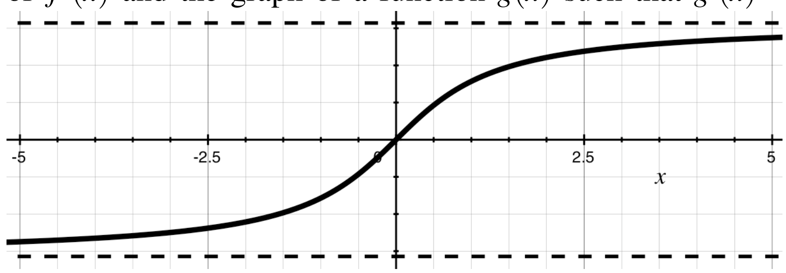
(3) Use the limit definition of the derivative to prove that the derivative of $f(x) = \sqrt{x}$ is $f'(x) = \frac{1}{2\sqrt{x}}$.

(4) Show that $f(x) = x|x|$ is differentiable at $x = 0$. (Hint: begin by rewriting f as a piecewise function.)

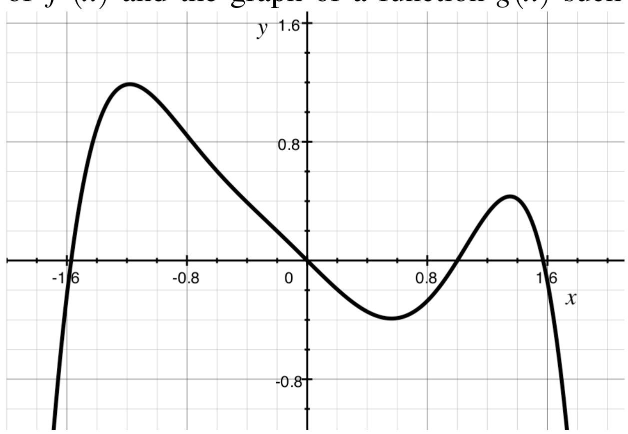
(5) Give a thorough explanation for why $f(x) = |x|$ is not differentiable at $x = 0$.

(6) Find the equation of the linear approximation to $f(x) = \sqrt{x}$ at $x = 4$. Use this to estimate $\sqrt{5}$.

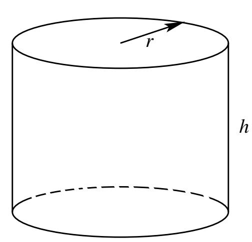
(7) Drawn below is the graph of a function $f(x)$. Sketch both the graph of $f'(x)$ and the graph of a function $g(x)$ such that $g'(x) = f(x)$.



(8) Drawn below is the graph of a function $f(x)$. Sketch both the graph of $f'(x)$ and the graph of a function $g(x)$ such that $g'(x) = f(x)$.



(9) Hormel is going to manufacture spam cans. Each can will be a cylinder of radius r and height h. The bottom and top of the can are made out of metal that costs 4 cents per square inch. The side of the can is made out of material that costs 2 cents per square inch. The can needs to hold 40 cubic inches of spam. What radius for the can make the cost of materials as low as possible? (Hint: the side of the can has area $2\pi rh$ and the top and bottom each have area πr^2. The volume of the can is $\pi r^2 h$.)



(10) River Phoenix is going to build a field bordering the Phoenix River. He has 2400 ft of fencing for three sides of the rectangular field and

will use the river as the fourth side. What are the dimensions of the field that has the largest area?

(11) Give complete, precise statements of the Extreme Value Theorem, Mean Value Theorem, and Intermediate Value Theorem.

(12) Draw the graph of a function with domain the interval $[0,1]$ that does not attain a global maximum or minimum on the interval.

(13) Draw the graph of a continuous function with domain the interval $(0,1)$ that does not attain a global maximum or minimum on the interval.

(14) Give the equation of a function that is continuous on the interval $[0,1]$ but does not have a derivative at $x = 1/2$.

(15) Give the equation of a bounded continuous function on the interval $(0,1]$ that does not have a limit as $x \to 0^+$. (The term "bounded" means that there is a number a such that $|f(x)| \leq a$ for all x in $(0,1]$.)

(16) Give the equation of a continuous function on the interval $(0,1]$ that does not attain a global maximum and also does not attain a global minimum.

(17) Use the Intermediate Value Theorem to explain why if f is a continuous function with domain equal to $[a,b]$ and range contained in $[a,b]$ then there exists c in $[a,b]$ such that $f(c) = c$.

(18) Give the main idea of how the Extreme Value Theorem can be used to prove the Mean Value Theorem.

(19) A car is travelling on a straight highway so that at time t (minutes) is is a distance of $t^2 - t + 1$ meters from home.

   (a) What is the average velocity of the car between $t = 2$ minutes and $t = 5$ minutes?
   (b) Find a t value so that the instantaneous velocity of the car at time t is exactly equal to its average velocity between 2 and 5 minutes.

(continued on next page)

(20) Find derivatives of the following functions:

(a) $f(x) = x^{\pi - e}$ (here π and e are the usual numbers)
(b) $g(x) = \frac{6}{\sqrt{x}} - \sqrt[3]{x^2}$
(c) $k(t) = 15t^3 + 2t^2 - \frac{1}{t}$

(21) For the following functions find, **using calculus**:

(i) All local maxima and minima in the specified interval
(ii) All global maxima and minima in the specified interval
(iii) All x values where the function is decreasing and concave down.
(iv) All x values where the function is decreasing and concave up.

(a) $g(t) = |3t - 4|$ for $-2 \leq t \leq 2$.
(b) $f(t) = t + 1/t$.