MA 121: HW 11 **Name:**______________________________

Answer these questions on a separate sheet of paper.

(1) In class we came up with a formula for the average value of a function $f(x)$ on an interval $[a,b]$. Explain how we did this.

(2) The ideal gas law says that if temperature is held constant, the pressure of a gas is inversely proportional to its volume. Suppose that at a volume of 2 cubic meters the pressure of the gas is 100 kiloPascals. As the volume changes from 2 cubic meters to .5 cubic meters, what is the average pressure of the gas? what is the average rate of change of the pressure of the gas? What is the instantaneous rate of change of the pressure of the gas (as a function of the volume)?

(3) Suppose that $f(x)$ is a positive function. Let S be the area between the graph of $y = f(x)$ for $0 \leq a \leq x \leq b$ and the x-axis. In class we came up with a formula (involving an integral) for the volume of the solid obtained by rotating S around the y axis. Using the same method find a formula for the volume of the object obtained by rotating S around the x-axis. (You could of course look up the formula. I'm asking you to come up with it on your own using a method similar to what we did in class.)

(4) The force exerted by a particle with charge +1 at the origin on another particle P of charge +1 a distance of r away is proportional to $\frac{1}{r^2}$.

(a) Calculate the work necessary to move P (in a straight line) from a distance of $r = 2$ to a distance of $r = 1$.

(b) Calculate the work necessary to move P from a distance of $r = 2$ to a distance of $r = s$ for some $0 < s$. Call this number $W(s)$.

(c) Find $\lim_{s\to 0^+} W(s)$. Explain what the result means in the language of physics.

(d) Find $\lim_{s\to\infty} W(s)$. Explain what the result means in the language of physics.

(5) A 100 foot long rope that weighs (1/3) lb per foot is hanging off a 60 foot tall building. One end of the rope is attached to the top of the building and the rope hangs straight down the edge of the building

so that 40 feet of rope is coiled at the base of the building. Calculate the work necessary to haul the rope to the top of the building (coiling it at the top as you haul it up). (Hint: Treat the 40 feet of rope coiled on the ground separately from the rope hanging down the side of the building.)

(6) Consider a cubical tank with sides of length 20 m^3. It is half filled with oil (which has a density of 800 kg/m^3). How much work is required to pump the oil out the top of the tank?

To answer this question, work through the following:

(a) Choose coordinates so that the bottom of the tank is at $y = 0$ and the top is at $y = 20$. We will begin by calculating the work necessary to pump a thin slab of oil out of the tank.

(b) Consider a slab (horizontal cross-section) of oil of thickness Δy and with the bottom of the slab at height $y = c_i$. What is the volume of this slab?

(c) What is the mass of the slab of oil in the previous part? Using Newton's second law (F = ma) calculate the force necessary to move the slab upward. (Recall that the force due to gravity is approximately $g = -9.8m/s^2$.) How much work is required to move this slab to the top of the tank.

(d) Write down a sum giving an approximation to the total amount of work necessary to pump the oil out of the tank.

(e) Realize that the total work required is a limit of the previous sum.

(f) Recognize the limit of Riemann sums as being an integral. Write down the integral and solve it using the fundamental theorem of calculus.

(7) Consider the conical tank pictured below that is filled to a height of 10 meters with oil. Calculate the work necessary to pump the oil out the top of the tank.