MA 121: HW 10 **Name:**_____

Answer these questions on a separate sheet of paper.

(1) Consider the differential equation $f'(t) = tf(t)$.

   (a) Find all positive functions $f(t)$ so that $f'(t) = tf(t)$. Your answer may have undetermined constants in it.
   (b) Find a function $f(t)$ satisfying the differential equation such that $f(0) = 2$.
   (c) Sketch the slope field for the differential equation. (You may use the computer if you wish)
   (d) Sketch your solution $f(t)$ to (b) on top of the slop field in (c).

(2) Explain the significance of the differential equation $f'(t) = k(f(t) - S)$ where k and S are constants and show how to find all functions satisfying the differential equation. (We did this in class, you are asked to work through those steps carefully and explain them.)

(3) Suppose that a cup of Darjeeling is at 200° at 3 PM and that 5 minutes later it is at 150∘. If the room is at 80°, how hot is the tea at 3:15 PM?

(4) Explain the significance of the logistic differential equation $f'(t) = kf(t)(M - f(t))$ and summarize the steps for finding all functions satisfying it. (We did this in class, you are asked to work through those steps carefully and explain them.)

(5) Suppose that in 1976 there is a population of 20 cats living in Meownneapolis, Meownisota. In 1976 they are increasing at a rate of 4 cats per year. Due to the mosquitos, the greatest the population of cats in Meownneapolis can possibly be is 5000. Approximately how many cats are in Meownneapolis in 2011? How fast is their population increasing in 2011?

(6) A 100 L tank of water is 5% salt. The tank is leaking water at a rate of .5 L/minute and freshwater is being added at a rate of .25 L/minute. Write down a differential equation representing the amount of salt in the water at time t.