MA 302: HW 3 additional problem

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem A: Let $\phi : \mathbb{R} \to \mathbb{R}^2$ be a parameterized curve. In class, we learned how to reparameterized ϕ to become a curve $\phi(s)$ so that for $0 \le s \le t$, $\phi(s)$ has length t. Let $k > 0$ be a constant. Show how to reparameterize ϕ to a curve $\phi(u)$ so that for $0 \le u \le t$, $\phi(u)$ has length exactly kt.

Problem B: Suppose that at time t a circle of radius ρ is tangent to the parameterized curve $\phi(t)$ where $\phi : \mathbb{R} \to \mathbb{R}^2$. What is a parameterization for the path taken by the center of the circle? (There are two possible answers, depending on which side of the curve the circle lie.)

Problem C: Suppose that at time $t = 0$, a circle of radius ρ is tangent to the parameterized curve $\phi(t)$. The circle rolls along the image of $\phi(t)$ in such a way that at time t, the circle is tangent to $\phi(t)$. Let P be the point on the circle directly to the right of the center of the circle at $t = 0$. In this problem, you will find a parameterization of the path $\mathbf{x}(t)$ taken by P.

(1) Find a parameterization $\mathbf{c}(t)$ for the path taken by the center of the circle. (Hint: use your work from problem B above.)
(2) Find coordinates for $\mathbf{x}(t)$ in the tangent space $T_{\mathbf{c}}(t)$.
(3) Find coordinates for $\mathbf{x}(t)$ in the usual coordinate system.
(4) For extra-credit animate the rolling circle and the path traced by P for the curve $\phi(t) = (\cos 3t, \sin 2t)$ for $0 \le t \le 2\pi$ and for $\rho = 1/2$.

Problem D: This is the same problem as problem C, but instead of the circle being tangent to the image of $\phi(t)$ at time t, the circle is rolling along the image of $\phi(t)$ so that it makes 1 revolution per second.

(1) At time t seconds, how far along the image of ϕ has the circle rolled?
(2) Find a reparameterization $\psi(t)$ of $\phi(t)$ so that at time t, the circle is tangent to the point $\psi(t)$. (Hint: Use problem A above)
(3) Find coordinates for $\mathbf{x}(t)$ in the usual coordinate system. (Your answer will likely have an inverse function in the expression – you won't be able to get it in closed form.)