**Problem Set 6**

**MA 111 Spring 2010**

Give complete and thorough answers to these problems on separate sheets of paper. The assignment is due in class on April 21.

**Problem 1:** How many vertices will an $n$–dimensional cube have? Explain (using analogies with lower dimensions) why your answer is reasonable.

**Problem 2:** Suppose that a 5–dimensional cube has edges of length 3. For each of the following questions give an answer and an explanation based on analogies with lower dimensions. You may first want to think about these questions for a 4–dimensional cube, before tackling the 5–dimensional cube.

(a) How many edges (1-dimensional faces) will the 5–dimensional cube have?

(b) How many squares (2-dimensional faces) will the 5–dimensional cube have?

(c) How many cubes (3-dimensional faces) will the 5–dimensional cube have?

(d) How many 4-dimensional faces will the 5–dimensional cube have?

(e) What is the 5–dimensional volume of the cube?

**Problem 3:** Denote a point in $\mathbb{R}^n$ by $(x_1,\ldots,x_n)$. The cone in $n$–dimensions is the set of points $(x_1,\ldots,x_n)$ for which

$$x_n = \sqrt{x_1^2 + x_2^2 + \ldots + x_{n-1}^2}$$

Thus, for example a cone in 2-dimensions has equation:

$$\begin{aligned} x_2 &= \sqrt{x_1^2} \qquad \text{this can also be written as} \\ y &= \sqrt{x^2} \end{aligned}$$

(Notice that $\sqrt{x^2} \neq x$. Instead, $\sqrt{x^2} = |x|$.)

(1) Write down the formula for a cone in 3-dimensions

(2) Write down the formula for a cone in 4-dimensions

(3) To try to visualize a cube in 4 -dimensions we can take 3-dimensional slices. Write down the formula for a 3-dimensional slice of the cone in 4-dimensions obtained by taking the slice with $x_3 = 1$. Use a calculator or computer to graph this. (To use a calculator or computer, you will probably have to use $x$, $y$, $z$ in place of $x_1$, $x_2$, and $x_4$ respectively)