Problem Set 3

MA 111 Spring 2010

Give complete and thorough answers to these problems on separate sheets of paper. The assignment is due in class on March 5.

Problem A: Give a careful explanation, in your own words, of why D_n can always be generated by a reflection and a rotation. Be sure to address whether or not *any* rotation and reflection will generate D_n.

Problem B: Give a careful explanation, in your own words, of why D_n can always be generated by two reflections. Be sure to address whether or not *any* two reflections will generate D_n.

Problem C: In the following parts you are given subroups of $\mathbb{S}_5$, for each state how many symmetries are in the subgroup. You should be able to do this without writing down all of the symmetries in the subgroup, although you may do so if you wish. In any case, be sure to thoroughly explain your reasoning.

(1) $\langle [1 \to 2 \to 3 \to 4 \to 5 \to] \rangle$

(2) $\langle [1 \leftrightarrow 2][3 \leftrightarrow 4], [1 \leftrightarrow 2] \rangle$

(3) $\langle [1 \leftrightarrow 2], [3 \leftarrow 4 \leftarrow 5 \leftarrow] \rangle$

(4) $\langle [2 \leftrightarrow 3], [3 \leftrightarrow 4] \rangle$

(Continued on following page.)

Problem D: Here is a change for 5 bells (A, B, C, D, E) known as the "plain hunt".

A	B	C	D	E
B	A	D	C	E
B	D	A	E	C
D	B	E	A	C
D	E	B	C	A
E	D	C	B	A
E	C	D	A	B
C	E	A	D	B
C	A	E	B	D
A	C	B	E	D
A	B	C	D	E

(1) Write down all symmetries in $\mathbb{S}_5$ such that a round in the plain hunt can be obtained by starting with A B C D E and then applying the symmetry. Thus, for example, the symmetry $[1 \leftrightarrow 4][3 \leftrightarrow 5]$ should be in your list since D B E A C shows up in the plain hunt. (You may copy your answer from PS 2, if it is correct.)

(2) In PS 2, you convinced yourself that this collection of symmetries is a subgroup of $\mathbb{S}_5$. Now explain why this subgroup can be generated by two symmetries. (You should specifically state two symmetries which generate the subgroup.)

(3) Explain how you know that this subgroup cannot be generated by a single symmetry.

Problem E: You are handed a sliding block puzzle with numbered blocks in the following pattern. The lower right hand space does not have a block. You are asked to solve the puzzle. Explain why this cannot be done.

1	2	3	4
5	6	7	8
12	9	10	11
13	14	15	