Problem Set 2

MA 111 Spring 2010

Give complete and thorough answers to these problems on separate sheets of paper. The assignment is due in class on February 22.

Problem A: In this problem you will show that the group D_3 is "the same as" the group $\mathbb{S}_3$. To do this, we must show that every symmetry of 3 points can be thought of as a symmetry of an equilateral triangle and that every symmetry of an equilateral triangle can be thought of as a symmetry of 3 points.

(1) Draw an equilateral triangle and write down its 6 symmetries (3 reflections and 3 rotations). Be sure to explain your terminology or notation (eg. are you rotating clockwise or counterclockwise).
(2) Write down the six symmetries in $\mathbb{S}_3$. (For example, $[1 \to 2 \to 3 \to]$.)
(3) Match the symmetries of the triangle with the symmetries in $\mathbb{S}_3$. Your matching should respect the group operation. For example, if S is matched with A and if T is matched with B then $S \circ T$ should be matched with $A \circ B$. To solve this problem, you will want to have a method for creating the matching. (Hint: number the vertices of the triangle.)

Problem B: Is it possible that D_n is the same as $\mathbb{S}_n$ for $n \geq 4$? Why or why not? (Hint: how many symmetries does each contain?)

Problem C: Here is a change for 5 bells (A, B, C, D, E) known as the "plain hunt".

A	B	C	D	E
B	A	D	C	E
B	D	A	E	C
D	B	E	A	C
D	E	B	C	A
E	D	C	B	A
E	C	D	A	B
C	E	A	D	B
C	A	E	B	D
A	C	B	E	D
A	B	C	D	E

(1) Using the notation of symmetries in $\mathbb{S}_5$, carefully describe the pattern of cross changes in the plain hunt.
(2) Write down all symmetries in $\mathbb{S}_5$ such that a round in the plain hunt can be obtained by starting with A B C D E and then applying the symmetry. Thus, for example, the symmetry $[1 \leftrightarrow 4][3 \leftrightarrow 5]$ should be in your list since D B E A C shows up in the plain hunt.
(3) Explain why the list of symmetries you wrote down in the last part is a subgroup of $\mathbb{S}_5$. To check closure, you shouldn't check all possible ways of two symmetries in your list, just give a general argument.

Problem D:

(1) Obtain a digital camera. (If you like, you may check one out from Media Resources; see the link below.) The web address for checking out a digital camera is:
`http://www.colby.edu/administration_cs/its/resources/media/rqpool.cfm`

(2) Take 5 photos of different objects or situations that demonstrate some type of symmetry.

(3) Pick your best two and briefly describe the symmetry involved. (Is it spatial symmetry? is there reflectional, rotational, or translational symmetry? is the only symmetry bilateral symmetry?) You should also mention if the symmetries differ if you take decorations (for example, color) into account.

(4) Change the name of the file on the photo to be of the form Last Name_Location.jpg . For example, if I took a photo at Colby's chapel I would name my file `Taylor_ColbyChapel.jpg` .

(5) Email both photos and your brief descriptions to sataylor@colby.edu. The subject line of your email should be "PS1 Photos".

Any people appearing in the photo (unless in a crowd or unrecognizable) need to provide their permission to be photographed. All photos should be tasteful and appropriate.