Exam 2 Study Guide

MA 111 Spring 2010

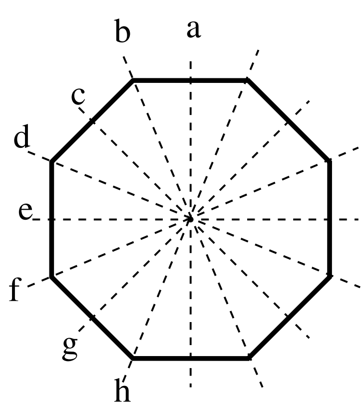
1. Symmetry

(a) Carefully explain why the 14-15 puzzle cannot be solved.

(b) The subgroup D_8 has 8 rotations and 8 reflections. The rotations are

$$\mathbf{I}, R_{45}, R_{90}, R_{135}, R_{180}, R_{225}, R_{270}, R_{315}.$$

The reflections are drawn below.



Let H be the subgroup $\{\mathbf{I}, R_{90}, R_{180}, R_{270}\}$. List all the distinct cosets of H in D_8.

(c) State LaGrange's Theorem and explain, in detail, why it is true.

(d) If H is a subgroup of a finite group G, and if g is a symmetry in G, give a precise definition of the coset of H in G containing g.

(e) Use LaGrange's theorem to show that if a group contains a prime number of symmetries then the only subgroups are $\{\mathbf{I}\}$ and the whole group. Use this to show that any symmetry other than $\mathbf{I}$ will generate the group.

(f) Explain why a dodecagon (12-sided regular polygon) cannot be decorated so that the decorated object has exactly 5 symmetries (including the identity).

2. Flatland

(a) Know the basic plot and characters of *Flatland*. Know the basic features of life in Flatland and Flatland society (for example, the Law of Compensation.)

(b) Summarize the major points made by Jann and Smith, Berkove, Baker. Be able to articulate why they come to different conclusions about the conclusion of *Flatland*.

(c) Know the following names and something about their contributions or ideas: William Whewell, J.S. Mill, Janos Bolyai, Nicholai Lobachevsky, C.H. Hinton, J.H. Newman, Euclid.

(d) Know the structure of Euclid's Elements

(e) List examples of satire from *Flatland*.

3. The mathematics of higher dimensions

(a) Understand the mathematical description of n-dimensional space.

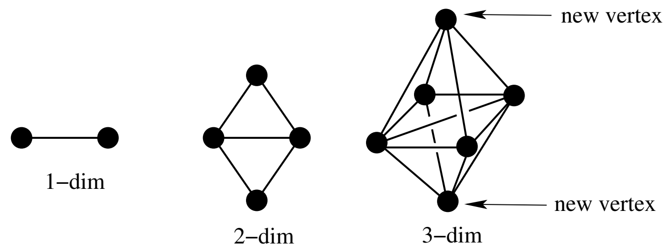
(b) Given an equation with n-variables be able to relate it to equations with fewer variables.

(c) Given an equation with n variables be able to take a slice of the higher dimensional object by plugging in constants for some of the variables.

(d) Given a pattern for creating objects in high dimensions, be able to use the pattern to count how many vertices, etc. it has.

Here is an example:

A 1–dimensional octotope is simply a line segment. A 2–dimensional octotope is a filled-in diamond created by introducing 2 new points (in a direction perpindicular to the line segment) and connecting each of the new points to all of the points on the 1–dimensional octotope. In general an n–dimensional octotope is created by starting with an $n-1$ dimensional octotope, introducing 2 new points in a perpindicular direction and joining each of the new points by line segments to each point on the $n-1$ dimensional octotope.



How many vertices does an n-dimensional octotope have? How many 2-dimensional triangles make up a 5–dimensional octotope? How many 3-dimensional octotopes make up a 5–dimensional octotope?

(e) Articulate why high dimensional geometry might be practically useful.

4. Probability

(a) Carefully state the problem of the points (i.e. the unfinished game discussed by Pascal and Fermat)
(b) Carefully define the following terms:
  (a) experiment (in probability)
  (b) sample space
  (c) probability space
  (d) event
  (e) uniform probability space
  (f) probability of an event E in a probability space.