Exam 1 Study Guide

MA 111 Spring 2010

1. Reading Questions

(1) Why do mirrors seem to reflect left/right but not up/down?

(2) What is the Ozma problem and how can it be solved?

(3) What is an *enantiomorph*? What are some examples from nature?

(4) What does it mean to say that parity is not conserved in weak interactions?

(5) What was "Mach's shock"? Why doesn't it give an example of asymmetry in physics?

2. Some math

(1) Give an example of a planar shape with non-trivial rotational symmetry.

(2) Give an example of a planar shape with non-trivial rotational symmetry, but no bilateral symmetry.

(3) Define the following groups or terminology:

group	even permutation
D_n	subgroup
$\mathbb{S}_n$	cycle number
transposition	odd permutation
C_n	generators for a group
braid group	identity in a group
associative property	inverse of a symmetry
generators	

(4) Is $\mathbb{R}$ (the real numbers) with addition a group? Is $\mathbb{R}$ with subtraction a group?

(5) Carefully explain why D_n can be generated by a rotation and a reflection.

(6) Carefully explain why D_n can be generated by two reflections.

(7) Carefully explain why $\mathbb{S}_n$ can be generated by transpositions.

(8) Carefully explain why each symmetry in $\mathbb{S}_n$ is either a combination of an even number of transpositions or an odd number of transpositions, but not both.

(9) Removed

(10) Explain why the symmetries [1 2] [3 4] and [2 3] [4 5] generate the subgroup of $\mathbb{S}_5$ corresponding to the plain hunt.

(11) Find all symmetries in the subgroup $\langle [1\,2\,3]\,[4\,5]\,,[4\,5]\rangle$ of $\mathbb{S}_5$.

(12) Show that R_{144} generates C_5

(13) How many symmetries are in the subgroup $\langle [1\,2\,3]\,[4\,5]\rangle$ of $\mathbb{S}_5$?

(14) How many symmetries are in the subgroup $\langle [1\,2\,3]\,[4\,5]\rangle$ of $\mathbb{S}_5$?

(15) Suppose that S and T are two symmetries in a group. Show that $(T \circ S)^{-1} = S^{-1} \circ T^{-1}$.

(16) Explain why D_3 and $\mathbb{S}_3$ are essentially the same group.

(17) Explain why D_n is a subgroup of $\mathbb{S}_n$ for all n.

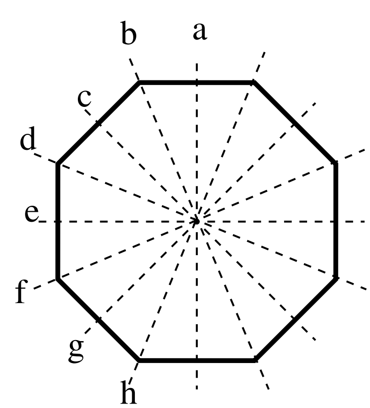
(18) Prove that D_n is not the same group as $\mathbb{S}_n$ if $n \geq 4$.

(19) If G is a group of symmetries of an object X and if H is a subgroup of G, give a method for decorating X so that H is the group of symmetries of the decorated object X. Explain why the method may not always work.

(20) The subgroup D_8 has 8 rotations and 8 reflections. The rotations are

$$\mathbf{I}, R_{45}, R_{90}, R_{135}, R_{180}, R_{225}, R_{270}, R_{315}.$$

The reflections are drawn below.



Perform the following computations

(a) $R_{45} \circ a$

(b) $a \circ R_{45}$

(c) $R_{135} \circ g$

(d) $R_{135} \circ g \circ R_{135}$

(21) Perform the following computations in $\mathbb{S}_5$:

(a) $[1\,2\,3]\,[2\,5\,4]$

(b) $[1\,2]\,[2\,3]\,[3\,5]\,[5\,4]$

(22) Let $g = [1\,2\,3\,4\,5\,6][7\,8\,9]$ in $\mathbb{S}_9$.

(a) Let $t = [2\,5]$. Draw convincing pictures to show how the cycle number of $g \circ t$ differs from the cycle number of g.

(b) Let $t = [4\,8]$. Draw convincing pictures to show how the cycle number of $g \circ t$ differs from the cycle number of g.

(23) Determine whether the following symmetries are odd or even permutations.

(a) $[2\,4\,6][1\,3]$

(b) $[1\,2\,3\,4][5\,6\,7]$

(c) $[2\,3\,9][1\,4\,8\,5][10\,11\,5]$

(d) $[2\,3\,4\,5][3\,4\,5\,6]$.