

MA 331: Topology

Spring 2009

The Basics

Class Times: MWF 9 – 9:50
Class Location: Keyes 103
Final Exam: Saturday, May 16 at 9:00 AM.
Textbook: *Elements of Combinatorial and Differential Topology*
by V.V. Prasolov. GSM 74. American Mathematical Society, 2006.

The Instructor

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The Subject

Topology is the loose affiliation of subjects sharing little in common but a *lingua franca*. At the undergraduate level, a course in topology most often consists of weeks upon weeks of glorified set theory culminating in one or at most two interesting theorems. This is not that course. The aim of this course is to give you a broad introduction to the objects most commonly studied by those mathematicians who call themselves TOPOLOGISTS. We do, however, need know at least some of the topological *lingua franca*. Thus, we begin with POINT-SET TOPOLOGY. We will proceed as quickly and as lightly as possible, skimming over the subject like an elf upon a snowfield. We then proceed to the study of SIMPLICIAL COMPLEXES. Along the way, we will prove the KURATOWSKI NON-PLANAR GRAPH THEOREM and the CLASSIFICATION OF SURFACES. We then turn our minds to ALGEBRAIC TOPOLOGY with a focus on the FUNDAMENTAL GROUP and COVERING SPACES. Finally, as time and energy allow we arrive at the subject of DIFFERENTIABLE MANIFOLDS. We will confine ourselves to two dimensional investigations, but what a lot there is to say! The list of topics might include MORSE FUNCTIONS, GEOMETRIC STRUCTURES, VECTOR FIELDS, and the GAUSS-BONNET THEOREM. Topology is a subject that is in constant conversation with other denizens of mathematics. Although no knowledge of these other disciplines is a prerequisite, we will not hesitate to make connections with geometry, group theory, differential equations, and analysis.

Such an ambitious trek will require energy and perseverance on your part. In return, you will see some very cool mathematics, not often glimpsed at an undergraduate level. This course, however, is not a graduate topology course. In graduate school one would see many of these same topics, but over the course of 2-3 semesters and at a much deeper level. For example, we will usually focus on topological creatures having fewer than three dimensions. In graduate school, you would do as much as possible without restricting the dimension. Additionally, we will usually avoid “pathological examples”, but it is often the case that one mathematician’s pathology is another mathematician’s way of life. In an effort to

compromise with those of a different mathematical bent, I relegate some pathological examples “to the exercises”.

Finally, I cannot stress enough how important it is that you devote considerable time and thought to this course. You will often be stuck and you will often need to rewrite your proofs. When you do have a proof it will often turn out to be incorrect. Such is the nature of mathematics, and topology in particular. For this reason, it is essential that you do the assigned reading (often IN ADVANCE of the class where the topic is covered) and work together. Parts of the course will be conducted by THE MOORE METHOD. This method has a long, venerable history in topology and is arguably the best way to learn parts of the subject. I will be with you in your struggles: if you need help or encouragement feel free to stop by.

The Grading

Class Participation	15 %	A	93% - 100%	D+	67% - 70%
Quizzes	10 %	A-	90% - 93%	D	63% - 67%
Homework	15 %	B+	87% - 90%	D-	60% - 63%
(excl. pt-set)		B	83% - 87%	F	< 60%
Point-set Homework	10 %	B-	80% - 83%		
Midterm Exam	20 %	C+	77% - 80%		
Final Exam	20 %	C	73% - 77%		
Final Paper	10 %	C-	70% - 73%		

However, achieving less than 50% of the combined homework and Final Exam points will result in an automatic F for the course. An A+ will be awarded to those students who show exceptional ability in and engagement with the course material. Such students will, for example, achieve close to 100% of the possible points in the course.

CLASS PARTICIPATION

You will frequently be asked to present mathematics in front of the rest of the class. Sometimes this may occur without advance notice. You are expected to be prepared to do so. You do not necessarily need to be completely correct in what you present, but you should be able to talk intelligently about the matter and it should be clear that you have given it considerable advance thought.

QUIZZES

Quizzes will last 5 minutes and will be given without warning. A typical quiz will ask you to regurgitate a definition that you are expected to know.

EXAMS

Exams will consist of a mixture of old and new problems. The new problems will be substantially easier than homework problems (assuming that you have done the homework). You will be given two weeks’ notice prior to the midterm exam.

“Point-set topology is a disease from which the human race will soon recover.” Poincaré

“If it’s just turning the crank, it’s algebra; but if it’s got an idea in it, it’s topology.” Lefschetz