

**Additional Problem on March 2, 2009**

Suppose that  $X$  is a topological space and that  $D$  is a topological space with the discrete topology. Let  $f: X \rightarrow D$  be a function. Prove that  $f$  is continuous if and only if for all  $x \in X$  there exists an open set  $U_x \subset X$  containing  $x$  such that  $f$  is constant on  $U_x$ . (In other words, for all  $z \in U_x$ ,  $f(z) = f(x)$ .)