**HW due April 20, 2009**

(1) Read pages 102-104 in text and pages 23-31, 35-39 of Maunder's *Algebraic Topology*.

(2) (a) Write down an abstract simplicial complex $K$, such that $|K| \cong S^1$ (i.e. is homeomorphic to the circle). As long as it is intuitively clear that you have done the right thing, you don't need to actually prove that the homeomorphism exists.
(b) Describe a map (continuous function) from $|K| \to |K|$ which is not simplicial.
(c) Describe a map from $|K| \to |K|$ which is simplicial.

(3) Suppose that $X$ is a contractible topological space and that $f\colon Y \to X$ is continuous. Prove that $f$ is homotopic to a constant map.

(4) Let $X$ be a topological space and let $p \in X$ be a point. Let $C$ denote the set of all continuous functions $f\colon I \to X$ such that $f(0) = f(1) = p$. The map $f$ is called a "loop based at $p$". Suppose that $f, g \in C$. Say that $f \sim g$ if and only if there is a "base-point preserving homotopy" connecting them. That is, there exists a continuous $F\colon I \times I \to X$ such that
(a) For all $s \in I$, $F(s,0) = f(s)$
(b) For all $s \in I$, $F(s,1) = g(s)$
(c) For all $t \in I$, $F(0,t) = F(1,t) = p$.
Prove that $\sim$ is an equivalence relation on $C$. The set of equivalence classes is denoted $\pi_1(X,p)$.

(5) A path-connected space $X$ is called **simply connected** if $\pi_1(X,p)$ has a single element. (Equivalently, if every loop in $X$ based at $p$ is homotopic by a base-point preserving homotopy to the constant map $f\colon I \to X$ where for all $s \in I$, $f(s) = p$.)
(a) Prove that a contractible space is path connected.
(b) Prove that a contractible space is simply connected.