PS 6 due May 8, 2009

Abelard and Heloise are playing games of "toss the toast". If the toast lands butter side up, Heloise wins. If the toast lands butter side down, Abelard wins. Unbeknownst to Heloise, Abelard has used lard instead of butter, making it more likely for the toast to land butter side down. In fact, the probability that the toast lands butter side down is 2/3.

(1) Explain what it means to say that the probability that the toast lands butter side down is 2/3.

(2) Assuming that the results of future tosses don't depend on the result of past tosses, what is the probability that Abelard will win 5 tosses in a row?

(3) Assuming that the results of future tosses don't depend on the result of past tosses, out of 5 tosses, what is the probability that Heloise will win exactly twice?

(4) Assume that the results of future tosses don't depend on the result of past tosses. Abelard and Heloise decide to play a game where the person to win the most tosses out of five tosses will win $100. They toss the toast 3 times and Heloise is ahead 2 to 1. Unfortunately, their dog, Ignatius, then eats the toast. What is the probability that Heloise would have won the $100?

(5) This problem continues the train of thought from the previous problem. What if instead of playing "the best out of 5 tosses", they played "first person to 3 wins"? (That is, as soon as someone wins 3 tosses, they stop playing) What is the probability that Heloise would have won the game? Be sure to explain your answer in relation to the previous problem.

(6) Abelard and Heloise find another piece of toast which Abelard again butters with lard. They decide to play the following game: They will toss the toast. If the toast lands butter side up, Abelard will give Heloise $99. If the toast lands butter side down, Heloise will pay Abelard $30. If they play this game many times, how much on average will Abelard win or lose? How much on average will Heloise win or lose?

(7) Heloise discovers that Abelard has buttered the toast with lard and demands that they switch to using a fair coin. (The probability of heads is 1/2 and the probability of tails is 1/2). Abelard and Heloise tossed the coin 15 times. Heloise always called Heads and Abelard always called Tails. Abelard wins the first 5 tosses. What is the probability that Abelard won 10 out of the 15 tosses?

(8) They decide to do this again. They toss the coin 15 times and Heloise wins 10 out of 15 tosses. What is the probability that Heloise won the first toss?