PS 5 due May 1, 2009

(1) Denote a point in $\mathbb{R}^n$ by $(x_1,\ldots,x_n)$. The cube in n–dimensions is the set of points $(x_1,\ldots,x_n)$ for which

$$\max(|x_1|,|x_2|,\ldots,|x_n|) = k.$$

for some number $k > 0$. The number k is called the radius of the cube.

(a) Write down the formula for a cube in 3 dimensions with a radius of 3

(b) Write down the formula for a cube in 4 dimensions with a radius of 1

(c) To try to visualize a cube in 4 -dimensions we can take 3-dimensional slices. Write down the formula for a 3-dimensional slice of the cube in 4 dimensions with radius 1. Take the slice with $x_4 = 1/2$.

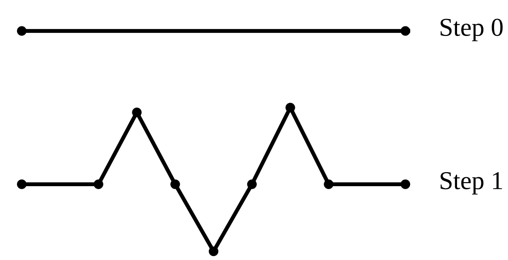
(2) Imagine creating a fractal by using the replacement rule pictured below. Step 3 would be obtained from step 2 by replacing each edge (between the round dots) with a zig-zag in exactly the same way that step 1 was created from step 0.

(a) How many edges does the nth step have? Be sure to explain your reasoning.

(b) The edge in step 0 has length 1. Each edge in step 2 has the same length. How long is each edge in step n? Be sure to explain your reasoning.

(c) What is the total length of the "curve" in step n? What happens to the length as $n \to \infty$?

(d) What is the fractal dimension of this curve (after infinitely many steps)?



(3) Imagine creating a fractal by starting with a square whose sides have length 1. Divide the square into 25 squares, each exactly the same size. Remove the middle square (leaving 24 squares). Now repeat this process on each of these smaller squares. After infinitely many steps we get a fractal I'll call the Sierpinski doormat. What is the fractal dimension of the Sierpinski doormat? Be sure to explain your reasoning.

(4) (Bonus) Imagine creating a fractal by starting with a square whose sides have length 1. Let k be an unknown odd number. Divide the square into k^2 squares, each exactly the same size. Remove the middle square (leaving $k^2 - 1$ squares). Now repeat this process on each of these smaller squares. After infinitely many steps we get a fractal I'll call the Sierpinski windowscreen. What is the fractal dimension of the Sierpinski windowscreen? Be sure to explain your reasoning. Your answer will be in terms of k. What happens to the fractal dimension as $k \to \infty$?