Problem Set 2

MA 111 Spring 2009

Complete the following problems on a separate sheet of paper. This assignment is due **Monday, March 2**.

Problem 1: Give a careful explanation, in your own words, of why D_n can always be generated by a reflection and a rotation. Be sure to address whether or not *any* rotation and reflection will generate D_n.

Problem 2: Give a careful explanation, in your own words, of why D_n can always be generated by two reflections. Be sure to address whether or not *any* two reflections will generate D_n.

Problem 3: In class we made a list of the 24 elements of $\mathbb{S}_4$. Using that list, perform the following computations.

(1) $[2 \leftrightarrow 3] \circ [1 \to 3 \to 4 \to 2 \to]$
(2) $[1 \leftrightarrow 2] \circ [2 \leftrightarrow 3] \circ [3 \leftrightarrow 4]$
(3) $[1 \to 2 \to 3 \to 4] \circ [1 \to 3 \to 2 \to 4]$

Problem 4: Do the symmetries $[1 \leftrightarrow 2] \circ [3 \leftrightarrow 4]$ and $[2 \to 3 \to 4]$ generate $\mathbb{S}_4$? Why or why not? If they do not, how many elements are in the subgroup generated by them?

Problem 5: In $\mathbb{S}_5$, consider the subgroup $H = \langle [1 \to 2 \to 3 \to 4 \to 5 \to] \rangle$. How many elements are in H?

Problem 6: Explain why we can think of the group D_n as a subgroup of $\mathbb{S}_n$.