Problem Set 1

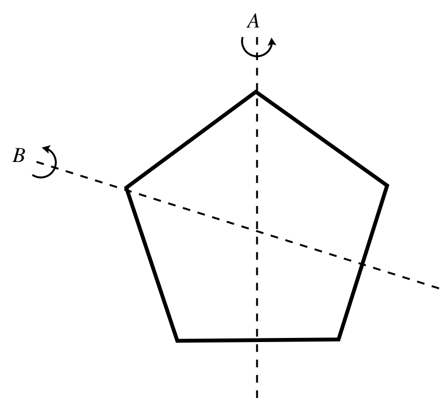
MA 111 Spring 2009

Complete the following problems on a separate sheet of paper. This assignment is due **Monday, February 16**.

Problem 1: Show that the set of rational numbers with the operation $+$ form a group. A rational number is a number which can be written in the form $\frac{a}{b}$ with a and b integers. (For example, $5 = 5/1$ and $-3/4$ are rational numbers. The number $\sqrt{2}$ is not a rational number.) You need to show four things: that the sum of two rational numbers is a rational number, that addition is associative (well, you can just assert this), that there is an identity element, and that every rational number has an inverse.

Problem 2: Explain why the set of real numbers with division as the operation is not a group. If use the set of real numbers except for zero and use division as the operation do we get a group? Why or why not?

Problem 3: Consider the pentagon in the plane drawn below. Two axes of reflectional symmetry have been drawn. They are labelled A and B. The pentagon also has rotational symmetry. Let R_θ denote a counterclockwise rotation by θ degrees. The rotations $\mathbf{I}$, R_{72}, R_{144}, R_{216}, R_{288} are all symmetries of the pentagon as well.



Each of the following combinations of A and B is actually a rotation. Determine which rotation it is. Be sure to show and explain your work.

(1) $A \circ B$
(2) $B \circ A$
(3) $A \circ B \circ A \circ B$
(4) $A \circ A \circ B \circ B$
(5) $B \circ A \circ B \circ A$

Problem 7: Suppose that G is a finite group and that g is an element of the group. Let g^n denote the result of combining g with itself n times.

(1) Show that there exist numbers $n \neq m$ such that $g^n = g^m$. (Hint, think about the set $\{g, g^2, g^3, g^4, \ldots\}$. Is this set finite or infinite?)
(2) Use part (1) to show that there exists a number $k \geq 1$ such that $g^k = \mathbf{I}$.

Problem 8: The circle has infinitely many rotational symmetries. Let $\mathbf{R}$ denote the rotational symmetries of the circle and let R_θ denote a clockwise rotation by θ degrees. Recall that $\sqrt{2}$ cannot be written as a fraction of whole numbers.

Show that there does not exist $k \geq 1$ such that $R^k_{\sqrt{2}} = \mathbf{I}$. In other words, show that if you rotate by increments of $\sqrt{2}$ degrees you will never return the circle to its original position. (Hint: If you rotate the circle by $\sqrt{2}$ degrees k times, by how many degrees have you rotated the circle?)

Problem 9:

(1) Obtain a digital camera. (If you like, you may check one out from Media Resources; see the link below.) The web address for checking out a digital camera is:
`http://www.colby.edu/administration_cs/its/resources/media/rqpool.cfm`
(2) Take 5 photos of different objects or situations that demonstrate some type of symmetry.
(3) Pick your best two and briefly describe the symmetry involved. (Is it spatial symmetry? is there reflectional, rotational, or translational symmetry? is the only symmetry bilateral symmetry?)
(4) Change the name of the file on the photo to be of the form Last Name_Location.jpg . For example, if I took a photo at Colby's chapel I would name my file `Taylor_ColbyChapel.jpg` .
(5) Email both photos and your brief descriptions to either MA111A@colby.edu or MA111B@colby.edu. The subject line of your email should be "PS1 Photos".

Any people appearing in the photo (unless in a crowd or unrecognizable) need to provide their permission to be photographed. All photos should be tasteful and appropriate.