Exam 1 Review

MA 111 Spring 2009

The exam will be in-class on Friday, March 13

- Why do mirrors reflect left/right but not up/down?
- What is the Ozma problem and how can it be solved?
- What is an *enantiomorph*? What are some examples from nature?
- What does it mean to say that parity is not conserved in weak interactions?
- What some examples of groups of symmetries?
- Give an example of a planar shape that has non-trivial rotational symmetry
- Define the following terms or notation:

group	monster group
D_n	subgroup
$\mathbb{S}_n$	coset
A_n	orbit of a point
C_n	stabilizer of a point
braid group	

- Is $\mathbb{R}$ (the real numbers) with addition a group? Is $\mathbb{R}$ with subtraction a group?
- Suppose that G is a finite group and that g is in G. Explain why there is a k so that $g^k = \mathbf{I}$.
- Carefully explain why D_n can be generated by a rotation and a reflection.
- Carefully explain why D_n can be generated by two reflections.
- Given the fact that $\mathbb{S}_3$ can be generated by transpositions, carefully explain why $\mathbb{S}_4$ can be generated by transpositions.
- Given the fact that $\mathbb{S}_4$ can be generated by transpositions, carefully explain why $\mathbb{S}_5$ can be generated by transpositions.
- State LaGrange's Theorem and explain, in detail, why it is true.
- State the orbit-stabilizer theorem and explain, in detail why it is true.
- Use the orbit-stabilizer theorem to determine an upper bound on the number of symmetries of a cube, tetrahedron, octahedron, icosahedron, and dodecahedron.
- Use LaGrange's theorem to show that if a group contains a prime number of symmetries then the only subgroups are $\{\mathbf{I}\}$ and the whole group. Use this to show that any symmetry other than $\mathbf{I}$ will generate the group.
- Write down all the elements of the following group without repetition

$$\langle S, T \quad | \quad S \circ T = T \circ S, \quad S^2 = \mathbf{I}, \quad T^3 = \mathbf{I} \rangle$$

- Be able to do calculations in $\mathbb{S}_n$ and D_n.
- Be able to use LaGrange's theorem to calculate the number of cosets of a given subgroup in a given group. For example, how many cosets of $\{\mathbf{I}, R_{180}\}$ in D_4 are there?
- Be able to write down all the cosets of a given subgroup in a given group. For example, write down all the cosets of $\{\mathbf{I}, R_{180}\}$ in D_4.