Euclid in the 19th Century

But Mr. Stelling took no note of these things; he only observed that Tom's faculties failed him before the abstractions hideously symbolized to him in the pages of the Eton Grammar, and that he was in a state bordering on idiocy with regard to the demonstration that two given triangles must be equal, though he could discern with great promptitude and certainty the fact that they were equal. Whence Mr. Stelling concluded that Tom's brain, being peculiarly impervious to etymology and demonstrations, was peculiarly in need of being ploughed and harrowed by these patent implements; it was his favorite metaphor, that the classics and geometry constituted that culture of the mind which prepared it for the reception of any subsequent crop. I say nothing against Mr. Stelling's theory; if we are to have one regimen for all minds, his seems to me as good as any other. I only know it turned out as uncomfortably for Tom Tulliver as if he had been plied with cheese in order to remedy a gastric weakness which prevented him from digesting it.

The Mill on the Floss George Eliot 1860

In the course of my law reading I constantly came upon the word "demonstrate". I thought at first that I understood its meaning, but soon became satisfied that I did not. I said to myself, What do I do when I demonstrate more than when I reason or prove? How does demonstration differ from any other proof?

I consulted Webster's Dictionary. They told of 'certain proof,' 'proof beyond the possibility of doubt'; but I could form no idea of what sort of proof that was. I thought a great many things were proved beyond the possibility of doubt, without recourse to any such extraordinary process of reasoning as I understood demonstration to be. I consulted all the dictionaries and books of reference I could find, but with no better results. You might as well have defined blue to a blind man.

At last I said,- Lincoln, you never can make a lawyer if you do not understand what demonstrate means; and I left my situation in Springfield, went home to my father's house, and stayed there till I could give any proposition in the six books of Euclid at sight. I then found out what demonstrate means, and went back to my law studies.

Abraham Lincoln

Euclid of Alexandria

- c. 300 BCE
- student of Plato's students
- started a school at Alexandria
- compiled the Elements from theorems of Eudoxus, Thaetetus, and others.

The Elements

Almost from the time of its writing and lasting almost to the present, the Elements has exerted a continuous and major influence on human affairs. It was the primary source of geometric reasoning, theorems, and methods at least until the advent of non-Euclidean geometry in the 19th century. It is sometimes said that, next to the Bible, the "Elements" may be the most translated, published, and studied of all the books produced in the Western world. -van der Waarden

http://www-history.mcs.st-and.ac.uk/Biographies/Euclid.html

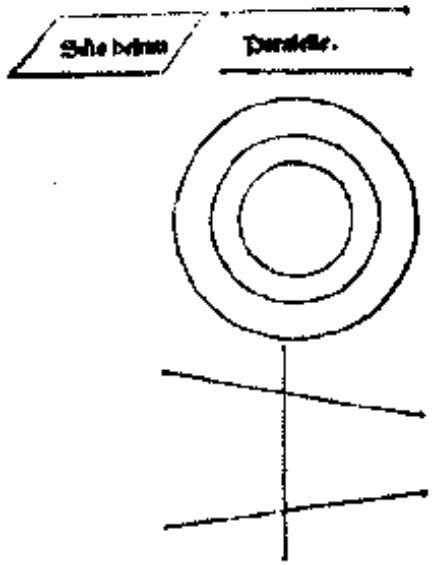
The structure of The Elements

Definitions

5 Postulates (Axioms)

5 Common Notions

Propositions



¶ Alia eſt ſimilis belimayum que oppoſita latera habet equalia atq3 oppoſitos angulos equales: idē tamen nec rectis angulis nec equis lateribus cōtinet: Preter has antē oēs quadrilatere figure belima/ripbe nominantur. ¶ Equidiſtantes linee ſunt que in eadem ſuperfi/cie collocate atq3 in alteramram partem ꝓtracte non conueniunt etiā ſi in infinitum ꝓtrahantur.

PEtitiones ſunt quinq3: ¶ A quolibet pūcto in quēlibet punctum rectā lineā ducere atq3 lineā definitā in cōtinuū rectūq3 quātūlibet ꝓtrahere. ¶ Super centrū quodli/bet quātumlibet occupando ſpacium circulū deſignare. ¶ Omnes rectos angulos ſibimmicem eſſe equales. ¶ Si linea recta ſup duas lineas rectas ceciderit duoq3 angulī ex vna par/te duob⁹ rectis angulis minores fuerint iſtas duas lineas in infinitū ꝓtractas ꝓcul dubio ꝓtractū ire. ¶ Duas lineas rectas ſuperficiē nul/lam concludere.

COmmunes animi ꝓceptiones ſunt hec: ¶ Que vni 7 eidē ſunt equalia 7 ſibiīmicē ſunt equalia: ¶ Et ſi equalib⁹ equa/lia addant tota quoq3 fiēt equalia. ¶ Et ſi ab equalib⁹ eq̄/lia auferant que relinquūt erūt equalia. ¶ Et ſi ab ineq̄li/bus equalia demas q̄ relinquūt erūt ineq̄lia. ¶ Et ſi ineq̄/libus equalia addas ipſa quoq3 fiēt inequalia. ¶ Si fuerint due res vni equales ipſe ſibiīmicem erūt equales. ¶ Si fuerint due res quaꝝ vtraq3 vni⁹ cuiuslibet3 fuerit dimidiū vtraq3 erit equalis alteri. ¶ Si ali/qua res aliam ſuperponat applicetq3 et nec excedat altera alteram: ille

The structure of The Elements

Definitions

Definition 1: A point is that which has no part.

Definition 15: A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure equal one another.

http://aleph0.clarku.edu/~djoyce/java/elements/bookI/

The structure of The Elements

Postulates

1. To draw a straight line from any point to any other point.

2. To produce a finite straight line continuously in a straight line.

3. To describe a circle with any center and any radius.

The structure of The Elements

Postulates

4. That all right angles equal one another.

5. That if a straight line falling on two straight lines makes the two interior angles on the same side less than two right angles then the two straight lines produced indefinitely meet on the side having the two angles which are less than two right angles.

The structure of The Elements

Common Notions

1. Things which equal the same thing also equal each other.
2. If equals are added to equals, then the wholes are equal.
3. If equals are subtracted from equals, then the remainders are equal.
4. Things which coincide with one another equal one another.
5. The whole is greater than the part.

The structure of The Elements

Propositions

10. To bisect a given finite straight line.

18. In any triangle, the angle opposite the greater side is greater.

http://aleph0.clarku.edu/~djoyce/java/elements/bookI/bookI.html#posts

Geometry in Victorian England

- Leibniz notation for Calculus allowed on Tripos in 1819.
- c 1819 geometry deemphasized. Reemphasized in 1848.
- 1839 mathematical suggestions by Gregory corr. to higher dimensions.
- Some criticism of detail of The Elements

Geometry in Victorian England

Two Questions:

1. Are the higher dimensions suggested by analysis and algebra geometric entities?

2. Can geometries other than Euclid's be conceived? Is Euclidean geometry adequate for spatial reasoning?

Geometry in Victorian England

- 1860s mathematicians begin to consider non-Eucl. geometries of Bolyai and Lobachevsky (1820s)
- 1866 Riemann publishes. Discussed by Sylvester (1869), Helmholtz (1870), Clifford (1874).
- Projective Geometry popularized by Cayley beginning in 1859

But you must note this: if God exists and if He really did create the world, then, as we all know, He created it according to the geometry of Euclid and the human mind with the conception of only three dimensions in space. Yet there have been and still are geometricians and philosophers, and even some of the most distinguished, who doubt whether the whole universe, or to speak more widely, the whole of being, was only created in Euclid's geometry; they even dare to dream that two parallel lines, which according to Euclid can never meet on earth, may meet somewhere in infinity. I have come to the conclusion that, since I can't understand even that, I can't expect to understand about God. I acknowledge humbly that I have no faculty for settling such questions, I have a Euclidean earthly mind, and how could I solve problems that are not of this world? And I advise you never to think about it either, my dear Alyosha, especially about God, whether He exists or not. All such questions are utterly inappropriate for a mind created with an idea of only three dimensions.

The Brothers Karamazov Dostoevsky

(1880)

http://gutenberg.net.au/ebooks07/0700061h.html

Higher Dimensions

Algebraic manipulations made it convenient to speak of n -dimensions for $n > 3$. (c. 1866)

Rarely was existence claimed for these higher dimensions.

Higher Dimensions

"The science [of Abstract Geometry] presents itself in two ways; - as a legitimate extension of the ordinary two- and three-dimensional geometries; and as a need in these geometries and in analysis generally. In fact, whenever we are concerned with quantities connected together in any manner, and which are, or are considered as variable or determinable, then the nature of the relation between the quantities is frequently rendered more intelligible by regarding them (if only 2 or 3 in number) as the co-ordinates of a point in a plane or in space: for more than three quantities there is, from the greater complexity of the case, the greater need of such a representation; but this can only be obtained by means of the notion of a space of the proper dimensionality; and to use such a representation we require the geometry of such a space." Cayley 1869

quoted in *Mathematical Visions* by Richards

Ultimately, all discussions about the nature of geometry are discussions about its foundations, whether they take place at the forefront of mathematical research or on the apparently mundane level of elementary teaching. This is nowhere clearer than in England in the 1870s and 1880s. during this period nearly all of England's mathematical enthusiasts were directly involved in considering the optimal form of elementary geometry textbooks.

Mathematical Visions Richards p. 163

1868: Wilson publishes alternative to The Elements. Heavily criticized by de Morgan and Dodgson (1879).

BOOKS I.—V.

CONTAINING

THE SUBJECTS OF EUCLID'S FIRST SIX BOOKS:

FOLLOWING THE SYLLABUS OF GEOMETRY PREPARED BY THE GEOMETRICAL ASSOCIATION.

BY

J. M. WILSON, M.A.

LATE FELLOW OF ST JOHN'S COLLEGE, CAMBRIDGE, AND MATHEMATICAL MASTER OF RUGBY SCHOOL.

FOURTH EDITION.

London:
MACMILLAN AND CO.
1878.

19th c. Philosophy of Science

Philosophical Nativists

True knowledge of the natural world is in some essentially innate

Empiricists

True knowledge of the natural world is drawn from experience

How do humans come to exact knowledge of absolute truth?

Philosophical Nativists

William Whewell (1794-1866):

- Math is descriptive
- Best for developing reasoning skills
- Math definitions tied to objects being described, not arbitrary
- Science proceeds by induction: successive generalization and testing

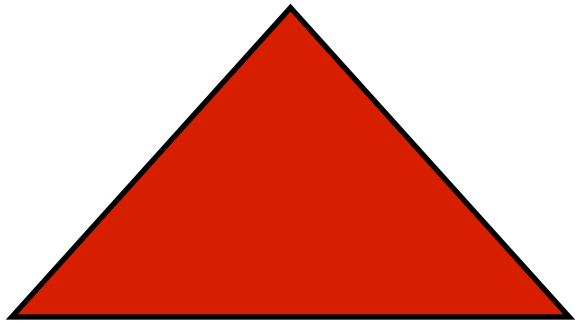
http://www.victorianweb.org/science/whewell.html

Philosophical Nativists

William Whewell (1794-1866):

There are:

- contingent truths: summaries of observed phenomena
- necessary truths: those whose opposites are inconceivable

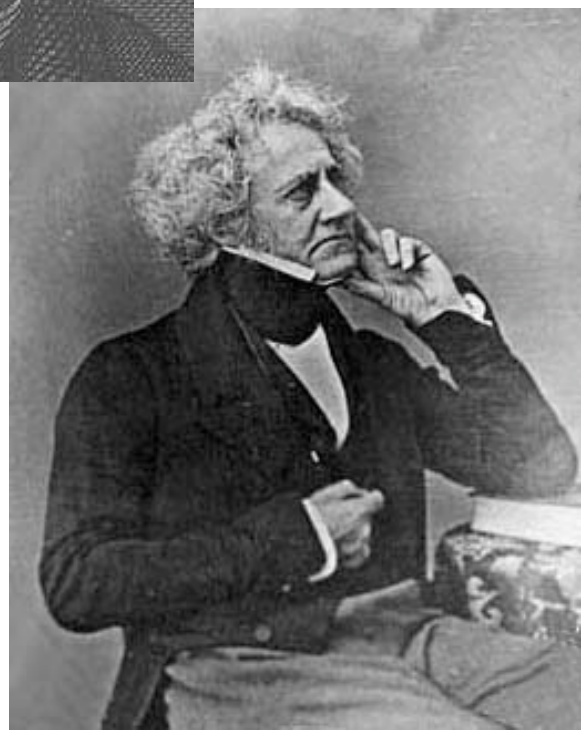


angle sum is 180°

http://www.victorianweb.org/science/whewell.html

Empiricists
John Herschel (1792-1871):

- Mind draws ideas from experience
- Mathematics describes external reality: space, number are objective facts.
- Induction in mathematics is natural and easy.



http://www-groups.dcs.st-and.ac.uk/~history/PictDisplay/Herschel.html

Empiricists

John Stuart Mill (1792-1871):

- Scientific reasoning about objective facts of nature
- Definitions neither true/ false - more/less acceptable descriptions
- Mathematics built upon definitions
- Rejects necessity of truth but agrees with descriptive view of mathematics

http://www.iep.utm.edu/m/milljs.htm

Baconian Induction

"THE DESCENT OF MAN

Man comes from a mammal that lived up a tree,
And a great coat of hair on his outside had he,
Very much like the Dreadnoughts we frequently see-
Which nobody can deny.

...

But this groping and guessing may all be mistaken,
And in sensitive minds may much trouble awaken,
So I'll shut up my book, and go back to my *Bacon*-
Which nobody can deny."

Blackwood's, 1871. Quoted in *Fact & Feeling* by Smith.

Francis Bacon

Novum Organum (1620)

Proceed by induction:
collect facts
a pattern eventually emerges
a hypothesis is formed

Whewell: Facts & Theories inseparable

de Morgan: Scientist interprets and writes laws of nature, is not a disinterested observer.

Abbott and Bacon

Abbott involved in "bitter dispute" over Bacon's personality

1876 Abbott supports Bacon's claims for induction. Bacon is the "prophet" who gives us a view of the "promised land of science".

1885 Abbott heavily criticizes Baconian induction.

"imagination ... is for Abbott the fundamental mechanism of both science and religious thought." -Jann

see Smith *Fact & Feeling*