## Fall 2022/MA 314 Study Guide for Exam 1

(1) Know precise definitions of the following terms:

(a) Metric space $(X,d)$

(b) Discrete, Euclidean, Manhattan, Paris, Max, Spherical, Hyperbolic metrics

(c) Isometry $f\colon X \to Y$ between metric spaces

(d) Open ball and open set in a metric space

(e) Continuous function $f\colon X \to Y$ between metric spaces

(f) Topologically continuous function $f\colon X \to Y$ between metric spaces

(g) The limit $(x_n)$ of a sequence in a metric space, and what it means for a sequence to converge.

(h) Sequentially continuous function $f\colon X \to Y$ between metric spaces

(i) Homeomorphism $f\colon X \to Y$ between metric spaces

(j) Compact metric space

(k) The length of a sequence in a metric space

(l) Complete metric space

(m) Surface

(n) Linear and Anti-linear fractional transform

(2) Be able to prove the following:

(a) A function $f\colon X \to Y$ between metric spaces is continuous iff it is topologically continuous iff it is sequentially continuous.
(b) If $f\colon X \to Y$ is a surjective continuous function and $X$ is compact, then $Y$ is compact
(c) If $f\colon X \to Y$ and $g\colon Y \to Z$ are continuous, then so is $g \circ f\colon X \to Z$
(d) If a finite length sequence in a metric space $X$ has a convergent subsequence, then it also converges.
(e) If $X$ is a compact metric space, then it is complete.
(f) Suppose that $U \subset \mathbb{E}^n$ has the property that there is a piecewise differentiable path in $U$ between any two points of $U$. Define the path metric on $U$ and prove that it is a metric.
(g) Prove that every isometry of $\mathbb{E}^2$ is the composition of translations, reflections, and rotations. If there are lemmas you need along the way, be able to state and prove them too.
(h) Prove that if $\tau$ and $\tau'$ are reflections of $\mathbb{E}^2$ about lines through the origin, then $\tau \circ \tau'$ is a rotation of $\mathbb{E}^2$ about the origin.
(i) Prove that every geodesic in $\mathbb{E}^2$ is a line segment.
(j) Prove that if $\rho$ and $\rho'$ are rotations of $\mathbb{E}^3$ around lines through the origin, then so is $\rho' \circ \rho$.
(k) Prove that horizontal translations, homotheties, and the standard inversion are all isometries of $\mathbb{H}^2$.
(l) Prove that every geodesic in $\mathbb{H}^2$ is either a vertical line segment or a portion of a circle centered on the $x$-axis in $\mathbb{E}^2$.