Geometry and Topology of Knots, Fall 2020 — Scott Taylor

Syllabus — For students, both in-person and remote.

What and Where

Course: The Geometry and Topology of Knots

Instructor: Scott Taylor

Email: sataylor@colby.edu – Don't hesitate to email me.

Office: Davis 207. I am happy to meet in person by appointment. Mostly office hours will be virtual. See below.

Course time: For in-person students, the course is 9 - 9:50 on MWF. Remote students should set their own time blocks for working on course material asynchronously. They should however also plan on meeting virtually with me.

Course Location: For in-person students: Davis 201. For remote students: a distraction-minimal location of your choosing.

Course Text: *The Knot Book* (2001 version) by Colin Adams. Published by the American Mathematical Society. An electronic version of the text may be purchases directly from the American Mathematical Society. Purchasing the book helps support the work of mathematicians all over the world. There will be additional readings and video provided via Moodle.

Course Materials: Located on the course webpage (`colby.edu/~sataylor`) and on the Moodle page.

Office Hours: I hope you come talk with me frequently about course material! Available times are listed on my webpage. All meetings with me must be scheduled in advance. It kills me to do that, because I love informal, spur-of-the-moment conversations, but it's necessary this semester both because of the health situation and because the local public school is on an every-other-day schedule. Follow these links to schedule meetings with me. After scheduling you'll be given zoom meeting information. It was also emailed to you at the start of the semester.

- MA 397 Virtual Office Hours.
- Individual Meetings or advising, discussing grades, etc. (Open to anyone, not just students in this course.)
- In-person meetings. Primarily, but not exclusively, for students in this course.

Academic Integrity

Academic Integrity is essential to the pursuit of truth and to gaining the most out of your studies. It is much more than just the absence of dishonesty. It is giving credit to all whose work you rely on; it is putting forth a good faith effort in the class; it is reaching out for help or encouragement or advice when you need it. It is doing the best you can in the circumstances you find yourself in. I value academic integrity greatly and expect you to. Don't hesitate to start a conversation with

me about it. It's a passion of mine. (And, yes, there are consequences for academic dishonesty. It doesn't matter what they are. What matters is if you know the difference between academic honesty and academic dishonesty and choose honesty. If you don't know the difference or if you are unsure – ask! If you need help – ask!) In our course, this plays out in two main ways: If someone (whether another Colby student or someone back home or someone on the internet) helps you out, give them credit. When you rely on other sources, whether books, articles, or websites – give them credit.

Related to this, but distinct, is the expectation that you not post course materials (videos, readings, classmates' work) to the internet (excepting of course your submissions to Moodle).

About the Course

Knots are central objects in geometry and topology and have profound connections to many other parts of mathematics. They are also beautiful, physical objects that show up both literally and abstractly in biology, chemistry, and physics. This course starts by studying knots through a hands-on, slightly informal method, known as "diagrammatic knot theory." We then move on to develop the essential and widely applicable theory of metric spaces (which has relevance throughout geometry, topology and analysis). We'll use metric spaces to gain a more precise and detailed understanding of knots and their connections to other mathematical areas. Along the way, we'll read some undergraduate-friendly research articles and explore some applications of knots to the sciences. Throughout the course, the emphasis will be on asking creative questions, formulating them mathematically, and taking steps to explore them rigorously.

Objectives for increasing mathematical maturity:

- Ask questions of varying degrees of sophistication about mathematical concepts, both concrete and abstract.
- Turn informal or imprecise ideas into precise mathematical concepts
- Tackle mathematical problems of unknown difficulty with creativity and courage
- Listen carefully to others discussing mathematics and ask constructive questions
- Give coherent written and verbal explanations of mathematics
- Engage in significant self-teaching of mathematics
- Be comfortable when encountering new mathematics
- Be able to express technical ideas in both formal and informal ways

Specific course content objectives:

- Using coloring invariants to distinguish knots
- Use polynomial invariants to distinguish knots
- Use continued fractions to distinguish 2-bridge knots

- Develop a thorough understanding of essential properties of metric spaces, as used in geometry and topology
- Use manifold theory to study knot invariants such as Seifert genus and bridge number
- Use metric spaces to make informal concepts such as knot equivalence and gluing mathematically precise
- Understand the beginnings of the relationship between hyperbolic geometry and knot theory
- Articulate the relationship between knot theory and certain problems in biology, chemistry, and physics
- Use knot calculators and SnapPy to find knot invariants

Other resources

Software: At various times you may like to use graphing software such as Mathematica or Desmos. Special software for knot theory includes the following free options:

(1) KnotInfo. An online database of knots and many of their invariants. (You will be required to use this)

(2) KnotAtlas. A wikipedia-like atlas of knots.

(3) SnapPy. This software allows you to draw a knot and perform sophisticated geometric and topological analyses of it, including calculating its hyperbolic volume (if it has one). (You may be required to use this, depending how far we get in the course.)

(4) Regina. This is software for analyzing surfaces in 3-manifolds.

(5) Seifert View. Software for visualizing Seifert surfaces (Windows only).

(6) KnotPlot. ($27.18). Software for making beautiful images of knots. (You are not required to use this)

Other Useful Texts: These are not required, but some lectures will be drawn from these books. If you want to explore knot theory more deeply, you may care to look at these.

(1) *An Interactive Introduction to Knot Theory* by Johnson and Henrich. This book is aimed at undergraduate and focusses on combinatorial approaches to knots.

(2) *Knots: Mathematics with a Twist* by Sossinsky. A friendly, very accessible book organized by historical developments in knot theory.

(3) *Knots and Links* by Cromwell. This is a friendly graduate level textbook, though many chapters are accessible to undergraduates.

(4) *Knots and Links* by Rolfsen. This is a friendly classic graduate level textbook, though some chapters are undergrad friendly. Hugely influential in the field.

(5) *Hyperbolic Knot Theory* by Jessica Purcell. A graduate level textbook, portions of which are accessible to advanced undergraduates.

(6) *Knots* by Burde and Zieschang. A standard reference for researchers in knot theory.

(7) *Knot Theory and Its Applications* by Murasugi. Another standard reference for researchers.

(8) *Algebraic Topology* by Hatcher. Freely available online. The standard graduate level algebraic topology textbook.

Assignments, Grading and such

Grading: Grading serves three main purposes – it provides extrinsic motivation for you to do important work, it enables me to give you feedback on your work, and it helps me to assign a rough marker as a measure of my estimate of your mastery of course material. The graded aspects of the course contribute to all three purposes, with varying degrees of emphasis. From your work over the semester, I'll assign a final course grade determined from the many parts as follows: 45% "doing" homework, 10% "asking homework", 15% midterm assessment, 15% final project, 10% question journal, 5% class participation.

Homework: Homework is assigned and collected three times per week. For both "in-person" and "remote" students, it is due prior to class. (Recall remote students choose their own class time.) It is to be turned in electronically to Moodle. That way if one of us sneezes on it, there's no risk. Homework can be hand-written and scanned (perhaps using an app on your phone), but it must be legible and there must be substantial space between problems, for me to write comments. If you type your work, I expect you to use LaTeX. *The expectation is that you are working on course material outside of class for 6 - 9 hours each week.* Each day's homework is broken up into 3 parts:

(1) **Absorbing.** This is where you do the assigned readings and watch any assigned videos.

(2) **Asking.** In this section, you will ask questions of varying levels of sophistication and imagination. These questions are compiled into your "question journal." More on this below.

(3) **Doing.** In this section, you will work examples and construct proofs. In many instances, you will be asked to spend a certain amount of time working on a proof and then reflect on what you accomplished and where you got stuck.

Your Asking homework and your Doing homework must be uploaded as separate PDFs. The file names must be in the following form (where DueDate is of the form MMDD)

- DueDate-LastNameFirstName-Asking.pdf
- DueDate-LastNameFirstName-Doing.pdf

For example, for homework due on August 26:

- 0826-TaylorScott-Asking.pdf
- 0826-TaylorScott-Doing.pdf

If you are taking the course remotely, either for the entire semester or for a short amount of time, you will need to upload the results (either in video or PDF form) of your classwork to Moodle. Follow the same conventions but appending "InClass". If you need to upload more than one item per class period use "InClassA" and "InClassB" etc. For example:

- 0826-TaylorScott-InClassA.pdf

I will make every attempt to give you feedback on your work within 24 hours, although my ability to do so is dependent on many factors, including my family responsibilities in the time of Covid-19.

Important Note: My time estimates for how long homework will take and the requirements for how long you are to work on a particular problem are under the assumption that you have "turned off" all distractions you have control over. (Namely: email, social media, etc.) In general, you will get more from the course the less time you spend on the internet and the more time you spend concentrating, reading, drawing, and writing.

Flexibility: Here are three distinct options for flexibility this semester. For any given situation you may invoke any one of these three options.

(1) **Sickness:** If you are sick, please email me to arrange for alternate due dates.

(2) **Weekly Emergency Clause:** At any time during the semester, by sending me an email, you may also invoke the Weekly Emergency Clause. Under the emergency clause, as long as you complete the "Absorbing Homework" and the "Asking Homework" you will receive an automatic grade of "73%" on the week's "Doing Homework" (without doing it). You should invoke the "Weekly Emergency Clause" if you have a significant disruption in your personal life which absorbs a good portion of your time and mental or emotional energy. In invoking the Weekly Emergency Clause, you do not need to tell me why you are invoking it, although I am happy to provide a listening ear if you want one. You may invoke the Weekly Emergency Clause as many times as needed.

(3) **Late Homework:** You really shouldn't get behind on the Absorbing Homework. Please! Doing so will severly limit your ability to get anything out of the course. Asking Homework and Doing Homework may be turned in up to 2 days late, for a 10% penalty.

Asking Homework: Each assignment will require you to ask questions concerning course material. I will give you feedback on your questions and expect to see future questions take into account the feedback I gave you on earlier questions. You will be required to share (some) of your questions in class or in an online forum with other students. You should maintain a folder dedicated to your Asking Homework. This folder is your "Question Journal." There are three kinds of questions you will be required to generate. To give you a sense for what I mean, I will provide examples drawn from a Calculus I course, but your questions will pertain to our course material.

- **Clarification Questions.** These are questions arising from confusion or unclarity about what is being said. For example: "Why does the definition of the derivative contain a limit?"

- **Digging Deeper Questions.** These are questions intended to push an underlying mathematical idea to the next level. For example: "What would go wrong if we define the derivative of f at a as $\lim_{x\to a}(x-a)/(f(x)-f(a))$ instead of $\lim_{x\to a}(f(x)-f(a))/(x-a)$? What would it measure?"

- **Inventive Questions.** These are questions that could potentially be the start of a research agenda. They do not involve minor variations of the material that is being presented to you, but provide scope for genuinely new ideas to appear. For example: "What is an example of a differentiable function whose derivative is not continuous?"

Your question journal will consist of all of your Asking Homework from the semester, together with a short, personal essay reflecting on the questions and your growth as a question-asking mathematician. More details about the essay will be distributed later.

Class Participation: For "in-person" students: your class participation grade is based on your willingness to participate in class, including sharing questions and ideas, going to the board, coming to (virtual) office hours. For "remote" students it is based on your regular meetings with me and the extent to which you stay on track with course assignments. You will receive periodic feedback about how you are doing with regard to class participation.

Midterm Assessment: This will consist of a individualized mini-project distributed in class Monday 9/28 that is due by the start of class on Wednesday 9/30. The assessment continues through class on Wed. 9/30. The "in-class" portion will consist of you reading another student's project and providing constructive comments on it. Your grade will be determined by a combination of my assessment of your work on the mini-project together with my assessment of the quality of the feedback you provide on another student's project. No part of your grade is determined by classmates' comments. Grading criteria will be distributed with the project. In essence, the goal is to draw on the questions you have generated and combine them with the techniques and ideas you've learned and the grade is determined by how thoroughly and effectively you do that. There is not necessarily the expectation that you completely solve any problem that you have generated.

Final Project: The final project will involve you choosing one or more questions that you have raised over the course of the semester and investigating them and keeping a record of your investigations. The grade will be based on the quality of your attempt, as well as a final essay and presentation describing your attempt. You will also be required to respond to one or more other students' final presentations. More details will come later. Depending on the question(s) you are tackling, there is likely no expectation that actually solve the problems you are working on.

Other useful things:

Religious Holidays Colby College is supportive of the religious practices of its students, faculty, and staff. The College is committed to ensuring that all students are able to observe their religious beliefs without academic penalty.

The College will enable any student to make up any course requirements scheduled during a religious holiday that is observed by that student. Students are expected to inform course instructors within two weeks of the beginning of the term of any religious observance that will conflict with coursework. The faculty member will then work with the student to find a reasonable accommodation that will allow the student to complete the academic work. In addition, no student will be required to participate in college events such as athletic commitments, lectures, or concerts on these holidays.

Personal Note: As a practicing Christian, I attempt to avoid working on Sundays. Emails I receive on Sunday are generally not answered until Monday. Not only is this a personal religious practice, it is an aspect of healthy living. I encourage you to establish, as much as possible, your own day of rest. If your schedule or workload does not allow you to set aside an entire day, I encourage you to be intentional about reserving a contiguous period of several waking hours for media-free rest, self-reflection, service or worship.

Learning Differences: Students with learning differences are encouraged to meet with me to discuss strategies for success. I am committed to helping all students succeed and to making reasonable accommodations for documented learning differences.

Sexual Misconduct/Title IX Statement: Colby College prohibits and will not tolerate sexual misconduct or gender-based discrimination of any kind. Colby is obligated to investigate sexual misconduct (including, but not limited to sexual assault and sexual harassment). If you wish to speak

confidentially about an incident of sexual misconduct, please contact Colby Counseling Services (207-859-4490) or the Director of the Gender and Sexual Diversity Program, Emily Schusterbauer (207-859-4093). Students should be aware that faculty members are considered responsible employees; as such, if you disclose an incident of sexual misconduct to a faculty member, they have an obligation to report it to Colby's Title IX Coordinator. Disclosure may include communication in-person, via email/phone/text, or through class assignments. To learn more about sexual misconduct or report an incident, visit http://www.colby.edu/sexualviolence/.