Geometry and Topology of Knots, Fall 2020 — Scott Taylor

Final Exam — For students, both in-person and remote.

This final exam as well as the final version of your question journal are due by **December 7 at 5 PM**. There is a place on Moodle for you to upload your exam as a single file. You may use the textbook, notes, course videos, other online material as long as you cite every source you rely on, apart from your own notes from class, the course textbook and course videos. You are also required to give credit to anyone (other than Scott) you talk with about the exam, even if they didn't explictly give you any ideas. Failure to cite your sources may result in a report of academic dishonesty. During the time period of the exam, you are not allowed to post questions pertaining to the course to online question-answering forums such as math.stackexchange, chegg, coursehero, etc. You are encouraged to not use random webpages for information. Your time is better spent thinking and writing, than searching, though you are required to do some searching on KnotInfo, the arXiv, and MathSciNet.

The syllabus originally stated that part of the final would involve responding to your classmates' work. However, due to the challenges of the semester, we won't do that.

I acknowledge I have read the rules and that I agree to abide by them.

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Signature

1. The Exam

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You have two options for the final exam. Email Scott your choice by **Tuesday, 11/24**. Notice that option (B) requires a bit more information.

(A) Choose one or more of your digging deeper or investigative questions and attempt to answer them. Your final exam will be a paper (which may be handwritten) that does the following:

(a) States a broad, but precise, overarching question about knots which draws on course material. (For example, is $u(K_1\#K_2) = u(K_1) + u(K_2)$?). The question may be phrased as a collection of related questions and may involve making preliminary definitions (for instance to define a new way of combining knots or a new potential knot invariant.)

(b) Lists what methods might be used to solve the over-arching problem, drawing on a variety of course material. You should describe with some care how the methods you list might be used to work on the problem.

(c) Lists 2 - 3 narrower, more specific versions of the overarching problem.

(d) Makes an intelligent attempt to apply course material to solve at least two of these narrower problems, drawing on KnotInfo and published papers and preprints as appropriate.

(e) Adapts the questions as appropriate to make them more tractable or provide avenues for further research.

(f) Explicitly references course material in a substantial and meaningful way.

(g) Makes multiple attempts to solve the problems, using multiple aspects of the course

(h) Communicates the results of the investigation in a clear, comprehensive, and compelling way, using illustrations as appropriate.

(B) With your request to Scott, **email** two aspects of the course you would like to potentially explore further (e.g. Jones polynomial/Seifert surfaces). He will email you one or two journal articles, which you should read and then write a paper (handwritten is okay) that does the following:

(a) Gives a through summary of the main results and methods of the paper(s).

(b) Carefully lists all the places where you do not understand the paper and carefully list clarification questions about the paper.

(c) Lists at least 4 digging deeper questions and at least 2 investigative questions inspired by the paper. These questions should explicitly reference course material and material from the papers. Each question may consist of several sentences and may include preliminary definitions or clarifying subquestions.

(d) For one of the digging deeper or investigative question, explain how the methods of the paper may be used to make progress on the question or why they can't be used to make progress.

(e) Use KnotInfo and MathSciNet to explore specific examples for the question to determine a potential answer. That is: What answer do you expect your question to have, and why? You are not required to work on actually solving the problem, just to get a sense for what the answer might be.

(f) Communicates all of the above in a clear, comprehensive, and compelling way, using illustrations as appropriate.

2. Grading Rubrics

The grading of this exam will likely feel more like the grading of a paper than the traditional grading of mathematics exam. That's okay! Not all mathematical knowledge can be captured or expressed in problem sets.

Rubric for Option A:

- (5%) Did you follow the directions?
- (25%) Effectiveness: Did you correctly identify useful methods for tackling your problem? Do those methods have a close connection to what was done in class? Did you use multiple approaches?
- (25%) Creativity: Do your questions go beyond those stated in the book or raised in class? Do they involve the invention of new mathematical ideas? Do they combine different course topics? Are the narrower, more specific versions of the questions well-connected to the bigger question?
- (25 %) Thoroughness of your Thoughts: Have you thought clearly about what you are saying? Do your answers reflect deep familiarity with course material? Have you pushed your ideas to the limits of what you could do based on what we've covered so far? Have you worked specific examples? Did you identify places where course material could give immediate benefit?
- (20 %) Communication. Have you clearly expressed your ideas in a way that is understandable to a classmate? Have you made effective use of diagrams? Did you give accurate citations as required?

Rubric for Option B:

- (5%) Did you follow the directions?
- (25%) Effectiveness: Did you give a full and accurate summary? Did you understand the portions of the assigned paper(s) that it is reasonable for you to understand based on course material? Did you identify the places that it is unlikely you would understand based on our course? Are your clarification questions coherently phrased and precise enough to admit an answer?
- (25%) Creativity: Do your questions go beyond those stated in the book or raised in class? Do they involve the invention of new mathematical ideas? Do they combine different course topics? Are the narrower, more specific versions of the questions well-connected to the bigger question?
- (25 %) Thoroughness of your Thoughts: Have you thought clearly about what you are saying? Do your answers reflect deep familiarity with course material? Have you pushed your ideas to the limits of what you could do based on what we've covered so far? Have you worked specific examples? Did you identify places where course material could give immediate benefit?
- (20 %) Communication. Have you clearly expressed your ideas in a way that is understandable to a classmate? Have you made effective use of diagrams? Did you give accurate citations as required?