MA 331: Topology	Fall 2018	Colby College

Course Location: Davis 217

Professor: Scott Taylor

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Office Hours: (subject to change)

Mon.	*2 – 3:30 PM*
Tues.	*9:30 – 11 AM*
Wed.	*1 -2:30 PM*
Thurs.	*12 – 1:30 PM*
Fri.	*1 – 2 PM*

and by appointment!

Office: Davis 207

Prerequisites: MA 274 or 275

Text: *An Illustrated Guide to Topology and Homotopy* by Sasho Kalajdzievski.
For further reading (you do not need to buy these):
Elements of Combinatorial and Differential Topology by V.V. Prasolov
Algebraic Topology by C.R.F. Maunder
Introduction to Topology by Bert Mendelson
Algebraic Topology by Allen Hatcher
Computational Topology: An Introduction by Edelsbrunner and Harer

The Course: Topology is the mathematical discipline which provides the formal context for discussing "continuity" in its broadest possible sense. Topology as a discipline can be subdivided into point-set topology, algebraic topology, geometric topology, and differential topology. Point-set topology provides the axioms, terminology, and basic results for all the other areas of topology, as well as many other parts of mathematics. However, most of the interesting results from topology arise in the other areas. Our goal this semester is to cover as much of point-set topology as is necessary to get to really interesting results and then to explore facets of algebraic, geometric, and differential topology. Additionally, topology is becoming an increasingly important tool in many areas of applied mathematics, computer science, and data analysis. We'll take a look at some of the ways topology is useful in these areas and you'll have the opportunity to explore more on your own in the final project.

Although we'll explore some applications and a bit of the computational side of topology, this course is based on theorems and proofs. These provide knowledge and understanding that is as certain as human endeavors can get. The ability to read, understand, and write proofs is a skill transferable to many other technical domains, both inside and outside mathematics. Computers can compute, but what is truly important in mathematics is the ability to convey technical knowledge and its consequences to a variety of audiences. Proofs are a concise, intellectually rigorous, way of communicating with other mathematically savvy people. You will also have the opportunity to convey mathematics by presenting at the board to your classmates and by writing an essay for a general audience.

Such ambitious goals will require energy and perseverence on your part. In return, you will see some very cool mathematics. I cannot stress enough how important it is that you devote considerable time and thought to this course. You will often be. When you do have a proof it may turn out to be incorrect. Such is the nature of mathematics, and topology in particular. For this reason, it is essential that you do the assigned reading (often IN ADVANCE of the class where the topic is covered) and work together. I will be with you in your struggles: if you need help or encouragement: stop by!

Objectives for increasing mathematical maturity:
By the conclusion of the course students will have improved in their ability to:
- Learn the basic language and results of point-set topology.
- Understand how algebra can be used to prove results in topology.
- Understand how topology is applicable.
- Engage in significant self-teaching of mathematics.
- Effectively communicate mathematics in both professional and in-
- formal styles.
- Write proofs in conventional mathematical style

Specific Course Content Objectives:
- Understand the definition, basic examples, and essential properties of metric spaces.
- Use the axioms of topology to prove theorems from first principles.
- Be able to prove results and construct examples concerning continuous functions, product spaces, quotient spaces, connectedness, and compactness.
- Understand the following major theorems of topology:
  – The Jordan Curve Theorem
  – The Classification of Surfaces
  – The Invariance of Domain and Dimension
- – The Brouwer Fixed Point Theorem
- Use the fundamental group to study topological spaces
- Use cut and paste methods to understand constructions in 2-dimensions.

Evaluation:
The numerical course grade will be a weighted average of the cumulative grades with weightings as follows:

15%/20%	minimum of Exam 1 and Exam 2
15%/20%	maximum of Exam 1 and Exam 2
20%	Final Exam
5%	Quizzes
5%	Reading Assignments and Class Participation
25%	Problem Sets
10%	Project

however, any student receiving less than 50% on all three exams will receive an F in the course.

Course letter grades will be assigned (subject to above caveat) according to the following scale. Any curve will be determined at the end of the course, according to the discretion of the instructor.

93 - 100 %	A	73 - 77 %	C
90 - 93 %	A-	70 - 73 %	C-
87 - 90 %	B+	67 - 70 %	D+
83 - 87 %	B	63 - 67 %	D
80 - 83 %	B-	60 - 63 %	D-
77 - 80 %	C+	below 60 %	F

Students who demonstrate an exceptional interest and ability in the course may be given an A+.

Academic Honesty
Academic honesty means that the work you present is your own, the ideas you communicate actually represent what you think and feel, and that you are upfront about your sources and inspirations. An act of academic dishonesty can either be intentional or unintentional. In either case, there are both informal and formal consequences. Being scrupulously honest is essential for the functioning of the college, for getting the most out of your own education, and for the success of mathematics as a discipline.

In this course, the greatest temptation towards academic dishonesty is presenting someone else's proof as your own (perhaps with minor modifications). If someone other than the TA or professor gives you the key idea for a proof, you must credit them in your write-up. *This includes any help you receive from online sources and applies even if you received help from a student who received help from a TA or the professor.* Acknowledging a source will never result in penalty. You are encouraged to work together and share ideas, but you may not copy another person's proof. Furthermore, you may not share LaTeX source code with other students and, in your papers/projects, you must give complete citations for any ideas, phrasings, or images you get from another source.

Academic dishonesty is a serious offense against the college. Sanctions for academic dishonesty are assigned by an academic review board and may include failure on the assignment, failure in the course, or suspension or expulsion from the College.

Learning Differences:
Students with learning differences are encouraged to meet with me to discuss strategies for success. I am committed to helping all students succeed and to making reasonable accommodations for documented learning differences.

Exams:
There will be two midterm exams on the evenings of **Thursday, October 11** and **Thursday, November 8** from 7 – 9:30 PM in **Diamond 122**. In the event that you are unable to attend the scheduled exam, you must let the instructor know *well in advance*.

The final exam is during exam period **2** on **Wednesday, December 12** from **1:30 – 4:30 PM**. It is college policy that the final exam cannot be rescheduled for personal convenience, *including airline reservations*. In the event that you have three or more final exams scheduled in close proximity, it is sometimes possible to reschedule one or more of them. See the registrar's webpage for details.

The three exams are cumulative, although the final exam is "more cumulative." The exams will present you with new mathematics and will expect you to prove theorems. One of the goals of the course is to equip you to succeed in such a situation. As with homework, your exam grade will be based not only on the correctness of your solutions, but also on your ability to clearly and coherently communicate them.

Daily Reading Assignments
Prior to each class you will need to read the textbook or other text and email me answers to the reading questions. The assignment and questions can be found in the weekly homework assignment, broken down by day.

Topology is a "definition-heavy" sub-discipline of mathematics. This means that you will need to both memorize a fair number of definitions and spend the time needed to internalize them. Doing this work before coming to class frees up class time for working through examples and discussing the significance of the ideas.

Class Participation
Asking questions, answering questions, sharing ideas, getting things wrong. All of these help you and your classmates learn. Topology is not just a set of facts but is also a way of seeing the world. You are expected to attend (and be on time for) class, to work with your classmates, and to ask and respond to questions. Since everyone is required to participate it is expected that you will allow your classmates the opportunity to do so and that all classroom interactions will be conducted with decorum and respect.

Problem Sets

Problems Sets will usually be collected on Mondays, but if you want to discuss the set with me on Monday, you may turn it in by 11 AM on Tuesday. This is in an effort to avoid having multiple problem sets for advanced math classes due on the same day. This does mean, however, that you will need to plan ahead. Although I will answer emails over the weekend, I will not be having weekend office hours.

Although Problem Sets will be collected weekly, the assignment sheet problems are broken out by day. You are *strongly* encouraged to do significant work on these problems prior to the day indicated.

Reminder: Whenever you make use of a source other than me or your textbook, you are required to cite it. This includes when classmates give you help or when you make use of something from the Internet. You are *strongly discouraged* from spending time looking for problem solutions online and you are *forbidden* from posting problems from the course to online help fora or using the textbook answer key to solve homework problems.

Project

Towards the end of the semester you will work on a project where you explore some aspect of topology not covered in the course. You will write a short paper and give a short presentation to the class on your findings. A schedule of deadlines and a project description will be distributed later.

A note on typing and printing mathematical documents: Not all web browsers correctly send PDFs with mathematical symbols to the printer. After you print any PDF (such as a portion of the textbook or a HW assignment) check to be sure that the symbols printed correctly. If not, open the PDF in Adobe Acrobat Reader or Preview (on a Mac) and print from there, rather than directly from the web browser.

If you type your work, unless otherwise specified, you are required to use LaTeX. Handwritten problem sets are fine.

Daily/Weekly HW requirements:

Writing Proofs

The strong proof will exhibit *careful logical reasoning* combined with *succinct expression* written with the inexorable drive of compelling mathematical ideas. A proof exhibits "careful logical reasoning" when each statement follows directly and immediately from a definition, axiom, or previously proved result, or a small number of these combined with one or more rules of logic. A proof exhibits "succinct expression" when illustrative examples are not used in place of careful logical reasoning, logical steps considered 'obvious' by the reader and which are not necessary, are omitted, and connecting words and transitions between sections while present are as brief and to the point as possible.

Guidelines for a successful assignment write-up[1]:

1. Your target audience is your classmates, NOT me or the grader! Your classmates should be able to read and understand your solution without having to ask you any questions about it.
2. You should use only full sentences, mathematical or otherwise. When using mathematical symbols convert them to words in your head to make sure that you are not missing appropriate verbs, articles, etc. Remember that mathematical language has a more rigid format (not unlike a programming language) than written English.
3. Writing many vague or repetitive sentences cannot be a substitute for saying the right thing once. Think your ideas all the way through before you shape them into steps of an argument. **Preliminary drafts are strongly recommended!**
4. I suggest placing consequent "steps" of an argument on separate lines, similar to the way this is done in instruction manuals, or in presentations of computer algorithms. At the very least, make good use of paragraph breaks.
5. Display longer formulas and diagrams on separate lines the way this is done in textbooks.
6. Skipping steps of an argument or asserting that something is "obvious" (henceforth called a "magic leap") can be considered a significant or even fatal error. I and the grader are the sole judges of what constitutes a magic leap or an insignificant error, so stay on the safe side and remember part 1.!
7. Do not put the proofs of two different theorems on the same piece of double-sided paper, unless both proofs are short and you are certain they will not need to be rewritten.
8. Your write-up should be laid out carefully and presented in a neat manner. Do a rough draft first, since a sloppy presentation will be penalized. The harder and less pleasant it is for us to follow your argument, the higher are the chances of you receiving a lower grade for the write-up.

[1] Adapted from guidelines used by Leo Livshits.

For example:

<u>1.7.4 Example</u>
Construct converses of the following statements:

1. If Elsie is a cow, then Elsie is a mammal.
2. If $x = 0$, then $x^2 = 0$.

Solution:

1. If Elsie is a mammal, then Elsie is a cow.
2. If $x^2 = 0$, then $x = 0$.

Here is an example of how to format the proof of a theorem:

<u>8.2.2 Theorem (Uniqueness of Inverses)</u>
Let x be a real number. There exists a unique real number y such that $x + y = 0$. Similarly, if x is not zero, there exists a unique y such that $x \bullet y = 1$.

<u>Proof:</u>

<u>Claim 1</u>: If x is a real number then there exists a unique real number y such that $x + y = 0$.

<u>Proof of Claim 1:</u> We must prove both that y exists and that it is unique. By Axiom 1 (pg 180), there exists an additive inverse y for x. That is, there exists y so that $x + y = 0$.

It remains to show that y is unique. To that end, suppose that y and z are both inverses for x. By the definition of additive inverse:

$x + y = 0$, and
(a.) $x + z = 0$.

Since each thing is equal to itself:
$y = y$.
Since 0 is the additive identity (Axiom 1), $0 + y = y$.
Consequently,
$0 + y = 0 + y$.
Thus, by equations (a.) we have:
$(y + x) + y = (z + x) + y$.
Since addition is associative (Axiom 1),
$y + (x + y) = z + (x + y)$.
Consequently, by equation (a.),
$y + 0 = z + 0$.
Since 0 is the additive identity (Axiom 1),
$y = z$.
Hence, additive inverses are unique as desired. Q.E.D (Claim 1)

<u>Claim 2</u>: If x is a non-zero real number, there exists a unique real number y such that $x \bullet y = 1$.

<u>Proof of Claim 2:</u> We must prove both that y exists and that it is unique. By Axiom 1 (pg 180), there exists a multiplicative inverse y for x. That is, there exists y so that $x \bullet y = 1$.

It remains to show that y is unique. To that end, suppose that y and z are both inverses for x. By the definition of multiplicative inverse:

$x \bullet y = 1$, and
(a.) $x \bullet z = 1$.

Since each thing is equal to itself:
$y = y$.
Since 1 is the multiplicative identity (Axiom 1), $1 \bullet y = y$.
Consequently,
$1 \bullet y = 1 \bullet y$.
Thus, by equations (a.) we have:
$(y \bullet x) \bullet y = (z \bullet x) \bullet y$.
Since multiplication is associative (Axiom 1),
$y \bullet (x \bullet y) = z \bullet (x \bullet y)$.
Consequently, by equation (a.),
$y \bullet 0 = z \bullet 0$.
Since 0 is the multiplicative identity (Axiom 1):
$y = z$.
Hence, multiplicative inverses are unique as desired. Q.E.D (Claim 2.)

Since we have proven both Claims 1 and 2, QED.

Other Sources:

- Pages 1 – 10 of these notes from a class by Donald Knuth: http://tex.loria.fr/typographie/mathwriting.pdf
- Sections 13-16 of this classic article by Paul Halmos: http://golem.ph.utexas.edu/category/2009/10/halmos_on_writing_mathematics.html
(Follow the obvious link to the article.)
- Chapter 0 of *Mathematical Proofs* by Chartrand, Polimeni, and Zhang.

Sexual Misconduct/Title IX Statement:

Colby College prohibits and will not tolerate sexual misconduct or gender-based discrimination of any kind. Colby is legally obligated to investigate sexual misconduct (including, but not limited to sexual assault and sexual harassment).

If you wish to speak confidentially about an incident of sexual misconduct, please contact Colby Counseling Services (207-859-4490) or the Director of the Gender and Sexual Diversity Program, Emily Schusterbauer (207-859-4093).

Students should be aware that faculty members are considered responsible employees; as such, if you disclose an incident of sexual misconduct to a faculty member, they have an obligation to report it to Colby's Title IX Coordinator. "Disclosure" may include communication in-person, via email/phone/text, or through class assignments.

To learn more about sexual misconduct or report an incident, visit http://www.colby.edu/sexualviolence/.

How to succeed in this course:

MA 331 is an intense course. I recommend the following to help you do your very best in the course and to get the most out of it.

1. Do some math everyday. Really.
Learning mathematics is a lot like learning to play an instrument, play a sport, or learn a language. You must practice everyday. The homework is intended to help you in your daily practice. But you should spend additional time each day reading the text and studying previous material. On average, you should spend 2 - 3 hours studying for each hour spent in class. In our case, that's at least 6 – 9 hours per week of homework and studying.

2. Participate in class. Yes, you.
Asking and answering questions is a great way to stay engaged with the material and verify for yourself that you know what's going on. The more people that participate, the more fun class is. I, and the rest of the class, value your questions and your answers, right or wrong. In fact, giving a wrong answer to a question is a

great way to learn the right answer. Make an effort to connect each class's activities to previous classes. Try to predict where the material will be going in the future. Take good notes and listen. If you can't do both, make a deal with a buddy: you take notes and they listen one day and the next day you switch. And, finally, volunteer to present in class. This is a great way to get feedback on your writing and proving.

3. Read the textbook.
   The lectures and the textbook will often present slightly different views on the same material. You will usually read the textbook before discussing the material in class. Think about the difference between the approaches and how they inform each other. After class, reread the section and see how your understanding has changed.

4. Form a study group.
   Introduce yourself to other people in the class and meet up outside of class to study and work on homework. The fourth floor of Mudd is a great place to meet in the evenings. Be sure that you don't copy answers, but learn from each other and then write the answers on your own. Compare lecture notes to be sure you copied everything correctly. Ask each other questions and explain material to each other.

5. Write your own problems.
   As you study for the exams, look back at all the examples done in class, in the text, or on homework. Try modifying them to make up your own problems. Try to solve your own problems – what makes your problems easier or more difficult than the ones you've seen before? Feel free to show me what you've done.

6. Be curious.
   Ask lots of questions. Ask questions to connect our course with material from other courses – especially writing, philosophy, and mathematics courses. Try to predict where the course is going. Try to create and prove your own theorems. Be curious – this is one of the privileges of being in college.

7. Visit me.
   I love working with students and I love to help you understand and appreciate the beautiful world of mathematics. Feel free to drop by, even when it's not my office hours. If I can't chat, I'll let you know. Ask crazy questions about the course. Ask questions about my research. Tell me about your past math experiences. Tell me about what subjects you love. Let me know when you have a concert or athletic event. Colby has a take-your-professor to lunch program where I get free lunch if you sign me in. Let's have lunch!

8. Spread the studying out over the semester.
   If you do math everyday, as suggested above, you won't have to cram for exams. You'll be able to sleep and, consequently, to think. You'll be happier and more

relaxed. You'll have time to write papers for your other classes. You'll have time to appreciate the New England winter. You don't need to pull all-nighters.

9. Have an exam strategy.
   For the midterm exams, you will have only 2 hours in which you have to do some significant mathematics. Be prepared to do some problems very rapidly and be prepared to think about others. If an example or proof was done in class or on homework, you should be able to repeat it very quickly on the exam. Know what you find difficult and what you find easy. Do the easy things first and then the difficult things. Write something for every problem. If you get stuck, tell me how you'd solve it if you could get unstuck. Figure out what the problem is testing and tell me what you know about that area. If the problem is too hard, rewrite it to make it easier. I love to give partial credit. Give me a reason to give you some. If you find yourself getting nervous: breathe deeply, remind yourself you've studied thoroughly, then figure out how to do the problem. Keep an eye on the time and don't spend too long on any one problem.