Fall 2018/MA 331 Final Project

One of the main goals of the Colby mathematics program is to produce graduates who are able to (and like to) learn mathematics independently and communicate their knowledge to a wide variety of audiences. In the final project for MA 331, you have a lot of flexibility as to topic and nature of the project. All projects will have a writing component and a presentation component. You should start by choosing from the following menu. As in a sandwich shop, be judicious about how you pair your choices. A lot of these projects are the mathematical equivalent of Big G's sandwiches: way too much to chew. So you should expect to address one tiny aspect of whatever it is you are working on.

(1) Choose a type of project:
   (a) Application of topology: Choose an application of topology in pure or applied math or another discipline and delve!
   (b) A big theorem or open problem: Choose a famously difficult theorem or open problem from topology and learn about some aspect of how people have tackled it.
   (c) Small open problem: Choose (with guidance) an open problem (probably one no one has thought about) and see how far you can get in solving it.
   (d) A big theory: Learn about some big topological theory we didn't talk about and learn the basics and something of what it is good for.
   (e) History: Explore the history of a topological idea.
   (f) Other: make a proposal.

(2) Narrow down the topic. Here are some ideas. Feel free to create your own.
   (a) Applications: digital topology, topological data analysis, knots in biology, topology and physics, geographic information systems, configuration spaces for robots, applications to music or abstract art.
   (b) Big theorems: Tychonoff's theorem, Tietze Extension Theorem, Topological conditions implying metrizability, Whitney's theorem on embedding manifolds in $\mathbb{R}^n$, 3D Poincaré Conjecture, Triangulation Conjecture, 4D (TOP) and 5D+ Poincaré Conjecture, Nielsen-Thurston Classification of homeomorphisms of surfaces, Poincaré-Hopf Theorem on vector fields, existence of exotic spheres, Nash embedding Theorem, Every 2-dim and 3-dim manifold can be triangulated.
   (c) Big open problem: Cabling Conjecture in knot theory, Slice-Ribbon Conjecture in knot theory, 4D smooth Poincaré Conjecture,
   (d) Small open problem: something about triangulations of surfaces, something about knotted θ-graphs in S^3. You should only expect to get started thinking about - not to actually solve it.
   (e) A big theory: Knot concordance, Homology theory, Cohomology Theory, Heegaard splittings, Corks in 4-manifolds, Morse Theory, Topological Quantum Field Theory (TQFT)
   (f) History: definition of "topology", origins of 3-manifold theory, connections to analysis, connections to differential equations, Moore method of teaching topology, connections between geometry and topology.

(3) Choose an audience for your writing assignment:
   (a) Students who have not yet had topology, but are happy learning some abstract mathematics
   (b) School children who don't have any of the vocabularly of abstract mathematics but like puzzles and fun ideas.
   (c) Classmates in your topology class
   (d) People who have taken several upper-level math classes.

(4) Find an introduction to the topic. This could be Wikipedia, an article in a Quanta Magazine, a popular math book, or introductory textbook.
(5) Choose a research level (or nearly so) published paper relevant to your project. I suggest using MathSciNet (ams.org/mathscinet) and the ArXiv. You must choose this carefully in consultation with me. I will likely recommend something to you. In all likelihood, ou definitely do not need to understand all or even most of the paper.

The Assignment Write a relatively short (< 10 pages) introduction or overview of your topic for your intended audience. Your paper or blog post must include both a big picture description, including why the topic is interesting, and also some carefully chosen details drawn directly from a published research (or near-research) paper. You must accurately cite the sources you relied directly on as well as include an "Acknowledgments" section giving credit to anyone or any source which influenced your paper. The length of your paper will depend heavily on choice of topic and audience. It is possible it will be only 2-3 pages. Quality is more important than quantity.

You must also give a 5 - 10 minute high level summary presentation to your classmates during the last two class periods. Basically, share with everyone what you worked on.

Assessment

As I grade, I will ask myself: Is material clearly presented, with an appropriate choice of detail? Is there a consistent choice of audience for the essay? Is the level of detail/rigor appropriate for that choice of audience? Are there an introduction and conclusion? Is the writing compelling and interesting? Will connect with the intended audience? Is it clear why the audience should be interested in the content being presented?

Deadlines

- Friday, November 16: Choice of project type and probable choice of topic. I am happy to help narrow things down.
- Tuesday, Novenber 20: Definite choice of topic and introductory resource chosen.
- Tuesday, November 27: Choice of (near-) research paper.
- Friday, November 30: Outline of paper and progress on writing
- Wednesday, December 5: Be prepared to give a summary of project to classmates.
- Monday, December 17, 11:59 PM: Final due date for paper. (Recall however, that there is a final exam: about which more later.) Absolutely no extensions will be given past this date.