Fall 2018/MA 331 HW 6: Feeling Connected

Before beginning this homework assignment, please review the guidelines for submitting homework. In particular, **If you consult a classmate or online source,** you must give credit for the help you received. Failure to do so may result in a report of academic dishonesty. You are, however, strongly encouraged to work with classmates – just be sure to give them credit for any ideas or help they provide!

Also, please write down the total amount of time spent working on the assignment at the top of what you turn in. If you are spending significantly more than 8 hours per week on homework assignments, you should talk with me to devise a new strategy.

The weekly homework assignments are broken out by day. It is crucial that you meet the deadlines for the reading assignments. When you do the reading, I encourage you to try to prove the theorems/propositions/etc. for yourself before reading the proofs in the book. As you read, sketch additional pictures, make marginal notes. In other words, be an active reader!

For the problems, I strongly encourage you to work with classmates, but be sure you are an active contributor to the discussion. Do not spend time looking for additional online sources. It is easy to waste a lot of time which could be used thinking. There are also a lot of proofs out there which are incorrect or which require a different background from what you have or assume that the course is structured differently.

---

1. For Friday

1.1. **Reading Assignment.** Read Sections 6.1 and 6.2. In Section 6.1, don't bother spending a lot of time on the proof that the reals and intervals in the reals are connected. In Section 6.2 you may skip Examples 4 and 5. Also, in the proof of Theorem 5, you may skip the part when I is infinite.

Answer the following questions in an email sent to me by Thursday night at 7 PM with the subject line "HW 6 Friday Reading Assignment." Memorize the definition of "connected" and "(connected) component".

(1) In class, we argued that the connectivity of $\mathbb{R}$ is equivalent to the IVT. In Section 6.1 and 6.2, the book develops a proof of these results somewhat differently. Which approach do you prefer and why?
(2) In Example 5 of Section 6.1, there is an odd example of a connected space. What is the similarity between this example and the topologists' sine curve from class?

1.2. **Standard Proofs and Problems (90%).**

(1) From Section 6.1: Problems 5, 6, 10.

(2) From Section 6.2: Problem 1.

1.3. **Advanced Proofs and Problems (10%).**

(1) Let X be a topological space. Define $\sim$ on X by $x \sim y$ if and only if x and y are in the same connected component of X. Give $X/\sim$ the quotient topology. What can you say about the connectedness and Hausdorfness of $X/\sim$?

(2) Do Problem 8 from Section 6.2.

2. For Monday

2.1. **Reading Assignment.** Read Sections 6.3 and 6.4 (but only until up to, but not including, the last paragraph on page 142.)

Answer the following questions in an email sent to me by Sunday night at 7 PM with the subject line "HW 6 Monday Reading Assignment." Memorize the definition of "path connected" and "path component" and "path".

(1) Summarize the difference between the properties of "connectedness" and "path connectedness"
(2) In Example 2 from 6.4, try to explain why the space isn't path connected. How does this example compare to the "topologists' sine curve"? Are the reasons for being non-path-connected similar or different?
(3) What lingering questions about connectedness or path connectedness do you have?

3. Proofs and Problems

No additional ones.