Fall 2018/MA 331   HW 5: Spacing out.

Before beginning this homework assignment, please review the guidelines for submitting homework. In particular, **If you consult a classmate or online source,** you must give credit for the help you received. Failure to do so may result in a report of academic dishonesty. You are, however, strongly encouraged to work with classmates – just be sure to give them credit for any ideas or help they provide!

Also, please write down the total amount of time spent working on the assignment at the top of what you turn in. If you are spending significantly more than 8 hours per week on homework assignments, you should talk with me to devise a new strategy.

The weekly homework assignments are broken out by day. It is crucial that you meet the deadlines for the reading assignments. When you do the reading, I encourage you to try to prove the theorems/propositions/etc. for yourself before reading the proofs in the book. As you read, sketch additional pictures, make marginal notes. In other words, be an active reader!

For the problems, I strongly encourage you to work with classmates, but be sure you are an active contributor to the discussion. Do not spend time looking for additional online sources. It is easy to waste a lot of time which could be used thinking. There are also a lot of proofs out there which are incorrect or which require a different background from what you have or assume that the course is structured differently.

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1. FOR WEDNESDAY

1.1. Reading Assignment. Read Sections 5.1 and 4.3. We covered much of 4.3 in class. In your reading of 5.1, focus on the how the definition of product topology shows up in the proofs.

Answer the following questions in an email sent to me by Tuesday night at 7 PM with the subject line "HW 5 Wednesday Reading Assignment." Memorize the definition of product topology.

   (1) Explain what the point of Proposition 2 in 5.1 is.
   (2) Which of the examples and illustrations in 5.1 were most helpful for understanding the product topology?
   (3) What questions do you have about the product topology?
   (4) Explain what it means to lue two subspaces together.

1.2. Standard Proofs and Problems (90%).

   (1) From Section 4.3: Problems 1, 7, 11.

   (2) From Section 5.1: Problems 1, 3.

1.3. Advanced Proofs and Problems (10%).

   (1) Let $\mathbb{R}$ be thought of as a group with addition as the operation. Let X be a topological space. A **flow** on X is the action of $\mathbb{R}$ on X. For $x_0 \in X$, the **orbit** of x_0 is the set $\{t \cdot x_0 : t \in \mathbb{R}\}$. (Here $t \cdot x_0$ is the result of the group element t acting on the x_0. The orbit of x_0 is **periodic** if there exists $P \in \mathbb{R}$ such that $(t + P) \cdot x_0 = t \cdot x_0$ for all $t \in \mathbb{R}$.

(a) Give an example of a flow on $\mathbb{R}^2$ such that no orbit is periodic. (Hint: Think about continuously translating in some direction.)

(b) Give an example of a flow on $\mathbb{R}^2$ such that every orbit is periodic. (Hint: Think about continuously rotating around some point.)

(c) Give an example of a flow on $T^2 = S^1 \times S^1$ such that every orbit is periodic. (Hint: Think about T^2 as the quotient space of $\mathbb{R}^2$ by the action of $\mathbb{Z}^2$ as in the previous homework.)

(d) Give an example of a flow on T^2 such that no orbit is periodic.

(e) For an arbitrary topological space X with a flow, prove that two orbits are either equal or disjoint.

(f) Come up with ideas for a proof, using the Brouwer fixed point theorem, that if a flow on $\mathbb{R}^2$ has a closed orbit, then there is a universal fixed point. That is, a point $a \in \mathbb{R}^2$ such that $t \cdot a = a$ for all $t \in \mathbb{R}$. You may be speculative and you should not try to actually complete your proof.

2. FOR FRIDAY

2.1. **Reading Assignment.** Read Section 4.4. Pay particular attention to Examples and less attention to the definition of CW-complex. Answer the following questions in an email sent to me by Thursday night at 7 PM with the subject line "HW 5 Friday Reading Assignment".

(1) What made the most sense in this section?
(2) What made the least sense in this section?
(3) What is a "cell"?

2.2. **Standard Proofs and Problems (90%).** From Section 4.4, do problems 2, 3, 12, 13.

2.3. **Advanced Proofs and Problems (10%).** From Section 4.4, do Problems 10 and 11.

3. FOR MONDAY

Nothing new.