Fall 2018/MA 331 HW 3:

Before beginning this homework assignment, please review the guidelines for submitting homework. In particular, **If you consult a classmate or online source,** you must give credit for the help you received. Failure to do so may result in a report of academic dishonesty. You are, however, strongly encouraged to work with classmates – just be sure to give them credit for any ideas or help they provide!

Also, please write down the total amount of time spent working on the assignment at the top of what you turn in. If you are spending significantly more than 8 hours per week on homework assignments, you should talk with me to devise a new strategy.

The weekly homework assignments are broken out by day. It is crucial that you meet the deadlines for the reading assignments. When you do the reading, I encourage you to try to prove the theorems/propositions/etc. for yourself before reading the proofs in the book. As you read, sketch additional pictures, make marginal notes. In other words, be an active reader!

For the problems, I strongly encourage you to work with classmates, but be sure you are an active contributor to the discussion. Do not spend time looking for additional online sources. It is easy to waste a lot of time which could be used thinking. There are also a lot of proofs out there which are incorrect or which require a different background from what you have or assume that the course is structured differently.

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1. For Wednesday

1.1. **Reading.** Read Section 3.5.

Memorize the definitions of: a function between topological spaces being **continuous at a point**, open mapping, closed mapping, homeomorphism, topology induced by a function.

Also memorize the following definition of continuity. This is equivalent, but different from the book's definition. It is more standard.

A function $f\colon X \to Y$ between topological spaces is **continuous** if and only if for every open set $U \subset Y$, the set $f^{-1}(U)$ is open in X.

In 3.5, the statements and proofs of Proposition 2, Proposition 3, and Proposition 5 are important examples of how to work with continuous functions in the setting of topological spaces. Try to prove the theorems for yourself before looking at the proofs in the book.

Email me answers to the following questions by 7 PM on Tuesday. The subject line of your email should be "MA 331: HW 3 WednesdayReading Assignment"

(1) Which proofs in 3.5 were hardest to understand? Which were the easiest?
(2) How is the definition of continuous function for topological spaces a natural generalization of the definition for metric spaces?

1.2. **Standard Exercises.** (90%) Exercises can generally be done by carefully putting together definitions or previous results. I suggest you use the definition of continuous function above for the following proofs, rather than the book's definition.

(1) From 3.5, do exercises 1, 2, 4, 5, 7

1.3. **Advanced Problems.** (10 %) Advanced problems usually require significant thought and creativity. You are not necessarily intended to solve them all. If you work on a problem, but don't solve it, you should explain what you've tried and how far you've gotten.

(1) From Section 3.5, do exercises 17, 18, 19, 20. The function r in problem 20 is called a "retraction." They are important in algebraic topology.

2. For Friday

2.1. **Reading.**

(1) Read Section 4.1: Memorize the definitions of: inclusion map, embedding,
(2) In preparation for 4.2, review the defintion of equivalence relation from MA 274. (If needed, you can refer to Section 1.1 from the course text or to my MA 274 book at web.colby.edu/sataylor)

The proof of Proposition 2 is a good example of how to work with the definition of subspace topology, as is Proposition 4.

2.2. **Standard Problems (85%).** Do exercises 1, 2 , 3, 6

2.3. **Advanced Problems (15%).** Do exercises 10, 14

3. For Monday

3.1. **Reading.** Read Section 4.2. Memorize the definitions of quotient map, quotient topology.

Pay particular attention to Examples 1 - 4 and the proof of Proposition 2. Email me answers to the following questions by Sunday 7 PM. The subject line of your email should be "MA 331 HW 3: Monday Reading Assignment"

(1) Why do we care about the quotient topology?
(2) What examples from the reading were most intriguing? What were difficult to understand?

No additional problems.