**Fall 2018/MA 331 HW 2: A convergence of minds has its limits.**

Before beginning this homework assignment, please review the guidelines for submitting homework. In particular, **If you consult a classmate or online source,** you must give credit for the help you received. Failure to do so may result in a report of academic dishonesty. You are, however, strongly encouraged to work with classmates – just be sure to give them credit for any ideas or help they provide!

Also, please write down the total amount of time spent working on the assignment at the top of what you turn in. If you are spending significantly more than 8 hours per week on homework assignments, you should talk with me to devise a new strategy.

The weekly homework assignments are broken out by day. It is crucial that you meet the deadlines for the reading assignments. When you do the reading, I encourage you to try to prove the theorems/propositions/etc. for yourself before reading the proofs in the book. As you read, sketch additional pictures, make marginal notes. In other words, be an active reader!

For the problems, I strongly encourage you to work with classmates, but be sure you are an active contributor to the discussion. Do not spend time looking for additional online sources. It is easy to waste a lot of time which could be used thinking. There are also a lot of proofs out there which are incorrect or which require a different background from what you have or assume that the course is structured differently.

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## 1. For Wednesday

1.1. **Reading.** Read the article *Rental Harmony* by Francis Su. You should read Sections 1 and 2 carefully. You don't need to read the other sections as carefully. My expectation is that you spend around 2 hours on this reading. If you have significant previous experience reading mathematics journal articles, you may not need to spend as much time.

Email me answers to the following questions by 7 PM on Tuesday. The subject line of your email should be "MA 331: HW 2 Wednesday Reading Assignment"

(1) What is a Sperner Labelling? What is the statement of Sperner's Lemma?
(2) Give a rough explanation of the proof of Sperner's Lemma in the 2-dimensional case.
(3) What ideas in the article intrigued you the most?
(4) What ideas in the article were most confusing?

## 2. For Friday

2.1. **Reading.** Read Sections 3.1 and 3.2. I encourage you to at least start the exercises below from 3.1 prior to reading 3.2. In 3.1, spend time understanding how each example of a topology satisfies the three properties in the definition of topology.

Memorize the definitions of: topological space, topology, open set, indiscrete topology, discrete topology, metrizable topological space, closed set, interior point, accumulation point, boundary point, Hausdorff.

In 3.2, the statements and proofs of Theorems 4, 5, 7, 8, and 9 are important. Try to prove the theorems for yourself before looking at the proofs in the book.

Email me answers to the following questions by 7 PM on Thursday. The subject line of your email should be "MA 331: HW 2 Friday Reading Assignment"

(1) Explain the way in which the notion of topological space generalizes the notion of metric space.
(2) Which of the examples in 3.1 were hardest to understand? Which were the easiest?
(3) In 3.2, which definitions were the easiest to understand? which the hardest?
(4) Answer the question at the end of Example 3 on page 41.
(5) Of Theorems 4, 5, 7, 8, and 9 in Section 3.2, which proofs were the hardest for you to follow? which the easiest? Were you able to devise any of the proofs yourself?

2.2. **Exercises.** (85%) Exercises can generally be done by carefully putting together definitions or previous results.

Note: If $f: X \to Y$ is a function and if $V \subset Y$ then $f^{-1}(V)$ is the notation for the set $\{x \in X : f(x) \in V\}$. It need not be the case that $f^{-1}$ is a function. In other words, $f$ need not be a bijection (or even injective). For example, if $f: \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = 0$ for all $x \in \mathbb{R}$, then $f^{-1}(V) = \mathbb{R}$ if $0 \in V$ and $f^{-1}(V) = \varnothing$ if $0 \notin V$.

(1) Do problems 2, 5, 6, 11 from Section 3.1.

(2) Do problems 2, 7,and 10 from Section 3.2.

2.3. **Advanced Problems.** (15 %) Advanced problems usually require significant thought and creativity. You are not necessarily intended to solve them all. If you work on a problem, but don't solve it, you should explain what you've tried and how far you've gotten.

(1) Do problems 6(a,b), 8, 16 from Section 3.2

## 3. For Monday

3.1. **Reading.** Read Section 3.3. Memorize the definition of: basis, second countable, coarser/finer. As you read pay particular attention to Thoerem 2 and its proof and Examples 6, 7, 8, 9, 10. Don't worry about Example 11.

Email me answers to the following questions by 7 PM on Thursday. The subject line of your email should be "MA 331: HW 2 Monday Reading Assignment"

(1) Of examples 6 - 10, which did you find most interesting?
(2) What from this section was easiest to understand? What did you have the hardest time with?

3.2. **Exercises.** (90%) Exercises can generally be done by carefully putting together definitions or previous results.

(1) Do problems 1, 2, 3, 8, 21 from Section 3.3.

3.3. **Advanced Problems.** (10 %) Advanced problems usually require significant thought and creativity. You are not necessarily intended to solve them all. If you work on a problem, but don't solve it, you should explain what you've tried and how far you've gotten.

(1) Do problems 4, 15, 19, 23