Fall 2018/MA 331 **Exam 2 Study Guide**

The exam will consist of a section of definitions, a section of old problems (eg. from Homework or class), and at least one new problem. Here is a list of problems (mostly ones you've seen before) to study. You should be pro-active about finding others to study, too.

(1) Suppose that X is path connected. Prove it is connected.

(2) Outline a proof that the topologists' sine curve is connected but not path connected.

(3) Suppose that X is (path) connected and that $f: X \to Y$ is continuous. Prove that $f(X)$ is (path) connected.

(4) Prove that X is disconnected if and only if there exists a surjective continuous function $f: X \to \{0,1\}$ (where the codomain has the discrete topology.)

(5) Suppose that X is a topological space and that $A \subset X$ is connected (with the subspace topology). Prove that if U, V is a separation of X then $A \subset U$ or $A \subset V$.

(6) Prove that if A and B are distinct connected components of a disconnected space X then there exists a separation of X separating A and B.

(7) Prove that the Cantor Set is totally disconnected. That is, the only connected components are singletons.

(8) Suppose that X is compact and that $A \subset X$ is closed. Prove that A is compact.

(9) Suppose that X and Y are compact. Prove that $X \times Y$ is compact.

(10) Suppose that X is compact and that $f: X \to Y$ is continuous. Prove that $f(X)$ is compact.

(11) Suppose that X is a compact topological space and that $A \subset X$ is an infinite subset. Prove that $A' \neq \varnothing$.

(12) Suppose that X is first countable and that $A \subset X$ with $A' \neq \varnothing$. Prove that A contains a sequence which converges in X.

(13) Suppose that X is second countable and sequentially compact. Prove that X is compact.

(14) Outline a proof of the fact that a subset of $\mathbb{R}^n$ is compact if and only if it is closed and bounded.

(15) Prove the extreme value theorem: If $f: X \to Y$ is continuous and X is compact, then there exist $m, M \in X$ such that for all $x \in X$,

$$f(m) \leq f(x) \leq f(M).$$

(16) Give an outline of the proof of the polygonal Jordan curve theorem.

(17) Give an outline of the proof of the Invariance of Dimension theorem.

(18) Prove that if X and Y are homeomorphic metric spaces, then then topological dimension of X equals the topological dimension of Y.

(19) Suppose that X is a topological space Prove that if X is disconnected and no connected component is compact, then the one-point compactification of X is connected. Prove that if X is disconnected and one or more connected components are compact then the one-point compactification of X is disconnected.