MA 274: Exam 1 Study Guide

Here are some suggestions for what and how to study:

(1) Know the definitions on the website from Chapters 1 - 5.7. Any other definitions/axioms that you need will be given to you.

(2) When you write a proof, focus on getting the organization clear and correct. If you have to skip some steps or make an assumption that you don't know how to prove, clearly state that that is what you are doing.

(3) Don't try to memorize proofs. Instead remember the structure of the proof (proof by contradiction, proof of uniqueness, element argument, etc.) and two or three key steps of the proof. Then at the exam recreate the proof.

(4) When studying, practice writing proofs, don't just look at them.

(5) Know the basic kinds of proofs: existence proofs, uniqueness proofs, element arguments, direct proofs, proofs by contraposition, proofs by contradiction. What are examples of each?

(6) Study the theorems we've proved in class and the more significant theorems from the homework.

(7) Be able to correctly write the negation of a statement.

(8) At the exam, leave time to write up a nicely written version of each proof. You should have enough time to sketch your ideas out on scratch paper before writing a final version of the proof.

(9) Here are some results you should study. You should also think about ways these problems might be varied. And you should study other problems too. On the exam axioms and definitions you weren't required to memorize would be given to you.

  (a) The number $\sqrt{2}$ is irrational.

  (b) There are infinitely many prime numbers.

  (c) There is no set U such that $A \in U$ if and only if A is a set. (Russell's Paradox)

  (d) The Halting Problem

  (e) DeMorgan's Laws

  (f) $A \cap (\bigcup_{\lambda \in \Lambda} B_\lambda) = \bigcup_{\lambda \in \Lambda} (A \cap B_\lambda)$

  (g) Suppose G is a group with operation $\circ$ and that $a \in G$. If $f, g \in G$ have the properties that $f \circ a = a \circ f = a$ and $g \circ a = a \circ g = a$, then $f = g$. (That is, the identity in a group is unique.)

(h) Suppose that G is a group with operation $\circ$ and identity $\mathbf{1}$. Let $a \in G$. If $f, g \in G$ have the properties that $f \circ a = \mathbf{1}$ and $g \circ a = \mathbf{1}$, then $f = g$. (That is, inverses in groups are unique.)

(i) Suppose that G is a graph and that a, b, and c are vertices. Then if there is a path from a to b and a path from b to c, then there is a path from a to c. (See the group project on graphs)

(j) The intersection of subgroups is a subgroup

(k) The intersection of convex sets is convex

(l) The intersection of event spaces is an event space.

(m) $X \times Y = Y \times X$ if and only if either $X = Y$ or one of X or Y is empty.

(n) If A and B are sets, then $A = B$ if and only if $\mathscr{P}(A) = \mathscr{P}(B)$.

(o) Remind yourself of the axioms of a metric space and suppose that X is a metric space with metric d. A subset $U \subset X$ is defined to be **open** if for every $a \in U$, there exists $r > 0$ such that the ball $B_r(a) = \{x \in X : d(x,a) < r\}$ is a subset of U. Suppose that $\mathscr{U}$ is a set such that if $U \in \mathscr{U}$ then U is an open set in X. Prove that the set $\bigcup_{U \in \mathscr{U}} U$ is open.

(p) Suppose that $\mathscr{E}$ is an event space and that for each $i \in \mathbb{N}$, V_i is an event. Prove that $\bigcap_{i=1}^{n} V_i$ is an event.