Graph Theory and Applications Final Project

Overview: You will select a topic pertaining to the course, read additional material pertaining to the topic, write a short paper summarizing the main ideas and then give a 15 minute presentation to the class on your topic.

Goals: So far this semester, we've seen a number of both theoretical and applied results from graph theory. This is your chance to either explore one of those topics in more depth or to explore a more advanced topic. You will also gain experience reading journal articles from mathematics or another discipline and presenting technical information.

Parameters:

- The choice of topic must be made in consultation with me, some ideas are listed below. The topics below are very broad; you'll want to quickly narrow to a single idea. I can provide sources for almost all of these topics.

- Your paper and presentation must make significant use of at least one journal article, must include at least one application to a field outside graph theory, and must include at least one significant graph theoretic result (though not necessarily its proof). You do not necessarily need to understand (or even) read the entire journal article.

- You may consult online sources such as Wikipedia, though the substance of your paper and presentation must come from other sources.

- You must carefully cite every source you use for your paper or presentation, using an inline reference to a bibliography/works cited page. I can show you how to easily do this using MathSciNet and LaTeX. You may also include a "Works Consulted" list for sources which did not have a direct impact on your final product but which were helpful in getting you there. A source you glance at only briefly need not be placed in the Works Consulted list.

  Citing our sources is a way of giving thanks to those whose work has influenced you and enables later scholars to retrace your intellectual journey. Additionally, it enables you and others to distinguish your contributions (perhaps only expositional!) from those of others.

- Your paper should be 5 – 10 pages long, though significant use of figures may make it longer. The target audience can be either your classmates or someone working in another field who may benefit from graph theoretic concepts. The grading rubric for the paper is attached.

- Your presentation should be 15 minutes long and is aimed at your classmates. It is likely a condensed version of part of your paper. You may like to use slides to enable rapid dissemination of figures or background ideas. The grading rubric for the presentation is attached.

Deadlines: You will not have additional homework while you are working on your project.

Tuesday, Nov. 21: Final decision on project topic.

Wednesday, Nov. 29: Outline of both paper and talk due in class.

Friday, December 1. Draft of paper due.

Dec. 4, 6, 8: In class presentations (to be scheduled later) 3 per day.

Friday, Dec. 8: Final version of paper due.

Presentation Grading Rubric:

(35%) Significant graph theoretic content beyond what was done in class.
(15%) Effective contextualization of the mathematics. Does the speaker motivate the ideas introduced? Does the speaker make connections with other subjects?
(25%) Clarity of presentation. Is the presentation understandable by the class? Is all relevant notation and terminology defined? Is effective use made of examples?
(25%) Preparation. Are visual aids well-designed? Is effective use made of the board? Has the talk been practiced?

Paper Grading Rubric:

(35%) Organization. Is the organization logical? Is there both flow and structure?
(15%) Motivation. Is clear why the reader should be interested in the topic of the paper?
(35%) Depth. Does the paper make significant use of at least one journal article? Does the paper explain connections to a subject outside of graph theory? Does the paper contain significant mathematics.
(15%) Mechanics. Are grammar, punctuation, spelling (mostly) correct? Are citations correct? Are quotation marks used appropriately?

Ideas for paper topics

You may suggest your own topic! But here are some ideas.

Note: You may not base your project on any project/paper/assignment/material from another course, though you are welcome to use the project to delve deeper into connections with another course.

- Independent Sets and Graph Homomorphisms.
  As an undergraduate Yufei Zhao solved a prominent open conjecture in graph theory concerning the number of independent sets in a *d*-regular graph. In a recent *Monthly* article, Zhao explains his solution and gives lots of references for understanding the significance of the problem.
- Algorithms. Code some of the more difficult algorithms from the course and investigate ways they've been developed or optimized by others.
- Random Graphs. A random graph is created by choosing a vertex set and then assigning edges between vertices according to some probability distribution. Investigate how the randomness interacts with various graph theoretic properties or explore how random graphs are used in mathematical modelling.
- Colorful Variations. Explore variations of the 4-color problem.
- Game Theory and the Chromatic Number. Coloring the vertices of a graph can be viewed in game theoretic terms and some of the theorems we've learned about can be reproven from that perspective.
- Graphs and Knot Theory. In knot theory, the Kauffman bracket is an exceptionally simple idea with profound consequences. It is calculated using recursion relationships similar to those in graph theory. Similary, other numbers associated to graphs can be turned into knot invariants.
- Graphs and thin position. (Probably just Claire or Muyuan should do this one.) Summer research students and I investigated ways of ordering the edges of graphs so that the ordering elucidates the geometric structure of the graph. In this project, you'll investigate relationships between our results and vertex coloring.
- Chromatic Number and Scheduling In this project you'll investigate scheduling algorithms related to the chromatic number of a graph.
- Percolation. Percolation is an idea from statistical physics in which quantities at one vertex are redistributed to other vertices. There are close connections to physics, chemistry, and probability theory.
- Small-World Networks. Many social networks are modelled using "small-world networks". You'll investigate what these networks are and what their properties are. You could perhaps begin with an article on the network of interactions between the characters in *Game of Thrones*.
- Random Walks on Graphs A random walk in a graph is a created by moving from one vertex to an adjacent vertex with some probability. One basic question is "What's the probability of returning to the original vertex?"
- Graphs and Games: Investigate how graphs are used to study board games (typically as Markov Chains)
- Constantine's Problem: A classic problem about how to distribute legions to maintain effective control over the empire. Fun and accessible.