

MA 397 Graph Theory Fall 2017
MWF 9 - 9:50 Davis 217

Professor: Scott Taylor
Office Hours: TBA
and by appointment!
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Prerequisites: MA 274, MA 253 or permission of instructor
Text: Douglas B. West *Introduction to Graph Theory*, 2nd edition

The Course:

Graph theory is the mathematical study of networks. Applications of graph theory are ubiquitous in physics, engineering, and computer science. This course introduces the basic terminology and results of graph theory; teaches how to construct rigorous arguments and useful examples; and develops the abilities to present mathematics both orally and in writing. Specific topics include euler and hamiltonian circuits, matching, connectivity and network flow, graph coloring, and algorithms. Particular attention is paid to applications.

Objectives for increasing mathematical maturity:

- Combine informal and formal reasoning to draw significant inferences about graphs.
- Combine theoretical and applied approaches in studying graph theory.
- Articulate the advantages and limitations of using graph theory to help solve problems from the natural and social sciences.
- Engage in significant self-teaching of mathematics.
- Effectively communicate mathematics in both professional and informal styles.
- Write proofs in conventional mathematical style.

Major Course Content Objectives

- Use graphs to model a wide variety of networks from the physical and social sciences.
- Understand the relationship between paths and cycles in graphs and fundamental properties of graphs.
- Use combinatorial methods to count the numbers of graphs having different kinds of properties.
- Use algorithms to construct different kinds of graphs or different kinds of paths in graphs.
- Write proofs using TONCAS¹ and extremal methods.
- Prove important theoretical results concerning Eulerian circuits, matching conditions, k -connectedness, network flows, and vertex colorings
- Independently investigate and report on results from an area of theoretical or applied graph theory.

¹“The obvious necessary condition is also sufficient”

Attendance: I value your involvement in the class, therefore class attendance is mandatory. Absence for official Colby activities requires prior approval. Absence for religious reasons will be considered excused if the policy in the college catalogue is followed. I reserve the right to take attendance. More than 3 recorded unexcused absences will result in the reduction by 1/3 of the final course grade. Excessive tardiness or early departure may also result in such a reduction.

Academic Honesty: Honesty, integrity, and personal responsibility are cornerstones of a Colby education and provide the foundation for scholarly inquiry, intellectual discourse, and an open and welcoming campus community. These values are articulated in the Colby Affirmation and are central to this course. You are expected to demonstrate academic honesty in all aspects of this course. If you are clear about course expectations, give credit to those whose work you rely on, and submit your best work, you are highly unlikely to commit an act of academic dishonesty.

Academic dishonesty includes, but is not limited to: violating clearly stated rules for taking an exam or completing homework; plagiarism (including material from sources without a citation and quotation marks around any borrowed words); claiming another's work or a modification of another's work as one's own; buying or attempting to buy papers or projects for a course; fabricating information or citations; knowingly assisting others in acts of academic dishonesty; misrepresentations to faculty within the context of a course; and submitting the same work, including an essay that you wrote, in more than one course without the permission of the instructors.

Academic dishonesty is a serious offense against the college. Sanctions for academic dishonesty are assigned by an academic review board and may include failure on the assignment, failure in the course, or suspension or expulsion from the College. For more on recognizing and avoiding plagiarism, see the library guide: libguides.colby.edu/avoidingplagiarism

Using the internet: The point of taking a course is to learn everything you can. Making excessive or inappropriate use of the internet prevents you from learning, damages my trust in you, and is discouraging to classmates who put forth their best efforts. In this course, you are permitted (though not encouraged) to make use of online resources which explain graph theory concepts, but not online resources which contain solutions to particular problems on homework or exams. In particular, you are not permitted to post course problems to websites (such as [math.stackexchange](http://math.stackexchange.com) or chegg.com) to solicit help from the denizens of the internet and you are not permitted to use solutions provided by such sites. In any case, many answers on such sites are incorrect - so you will waste time you could spend in office hours or thinking about the problem for yourself! You are also not permitted to make use of the solutions book to our text book. I would much rather you turn in a "solution" which is a discussion of all the things you tried and which didn't work than see a solution copied from the solutions manual. **In any case, any time you refer to any source - print or online - other than the course text, notes, or office hours, you are expected to cite your source.**

The Colby Affirmation

Colby College is a community dedicated to learning and committed to the growth and well-being of all its members.

As a community devoted to intellectual growth, we value academic integrity. We agree to take ownership of our academic work, to submit only work that is our own, to fully acknowledge the

research and ideas of others in our work, and to abide by the instructions and regulations governing academic work established by the faculty. As a community built on respect for ourselves, each other, and our physical environment, we recognize the diversity of people that have gathered here and that genuine inclusivity requires active, honest, and compassionate engagement with one another. We agree to respect each other, to honor community expectations, and to comply with college policies.

As a member of this community, I pledge to hold myself and others accountable to these values.

What does this mean to students?

- (1) We respect each other and ourselves.
- (2) We respect our physical spaces on campus.
- (3) We respect our academics and complete work honestly.

Evaluation: The numerical course grade will be a weighted average of the cumulative grades with weightings as follows:

- 20% Homework
- 5% Participation
- 15% Quizzes
- 10% Final Project and Presentation
- 15% Midterm Exam 1
- 15% Midterm Exam 2
- 20% Final Exam.

Caveat: earning less than 60% on each of the three exams will result in a grade of “F” for the course.

Course letter grades will be assigned (subject to the above caveat) according to the following scale. Any curve will be determined at the end of the course according to the discretion of the instructor. An A+ may be awarded to students who do exceptionally well and who demonstrate an exceptional interest in the course.

93 – 100 %	A	90 – 93 %	A-	87 – 90 %	B+	83 – 87%	B
80 – 83 %	B-	77 – 80 %	C+	73 – 77 %	C	70 – 73%	C-
67 – 70 %	D+	63 – 67 %	D	60 – 63 %	D-	below 60%	F

Homework: Homework is the most important part of this course. It is where you wrestle with new ideas, generate creative solutions, and convey your thoughts to others. You are expected to write complete, careful, and correct proofs. You should start the homework early, some problems will require multiple attempts and careful thought. If you are having substantial difficulty with a particular problem or the entire homework set, you should email me or come to office hours. I am eager to help you! You are encouraged to work with a partner on the homework, but all work should be your own. In other words, you may discuss particular problems but you may not copy someone else’s solution. Doing so is an act of academic dishonesty.

Structure and Schedule: Homework will generally be due on Fridays, although on occasion reading assignments may be collected on other days. Reading assignments and problem sets will always be posted on the course webpage. You are responsible for checking the webpage. If no homework

assignment is posted, you should refresh the webpage on your browser and, if that doesn't work, email me to let me know. In the special circumstance that there is no new homework, the webpage will make note of that. If you will not be in class on the day that homework is due you should arrange to turn it in at my office or to have a friend bring it to class. Late homework may be penalized. Start HW early!

Due to the small number of students and drastically different mathematical backgrounds, certain problems may be assigned only to certain students. If you are assigned a particular problem, you should take special ownership of it and be prepared to tell the class about it when the assignment is due.

Grading and Rewrites: Homework problems will be thoroughly graded for both mathematical correctness and completeness and clarity of the proof. Any proof for which you turned in an (attempted) solution on time may be rewritten within one week of its return to you. Rewrites are graded using the same criteria as the original and you have the potential to earn full credit on such problems.

Because there will be a lot of paper being passed back and forth between us, you must follow these guidelines for homework:

- Each assignment must have your full name, the date it is due, and the assignment number on each page of the assignment.
- Problems must be turned in in the order they are assigned.
- All solutions must be turned in on 3-hole punched paper and placed into the “official” homework binder.
- Each proof must be written on its own 2-sided sheet of paper. You are encouraged not to turn in your first draft of any of these problems.
- Rewrites must be turned in with the original solution. No credit will be given for rewrites without the original attached. It must be clear exactly which problems from which assignments are being rewritten.
- Homework must be very neat. This means: no messy scratchwork, no cramped writing, no huge eraser marks. If these guidelines are not followed you may be penalized. If you are incapable of writing neatly, you should type your solutions. \LaTeX is the most popular mathematical typesetting software, but you may also use software like *Scientific Word* or *Mathematica*. There are resources for learning \LaTeX on my webpage.

Proof Expectations: It is expected that every graded proof be written in conventional mathematical style. Each statement should follow logically and naturally from the previous ones. It is possible for a proof to be substantially correct but not to receive full credit because the writing was not up to standard. The audience you should keep in mind is an imaginary classmate who is not quite as “bright” as yourself. The danger in communicating mathematics is always assuming your audience knows more than they do! This being graph theory, good illustrations can help you communicate the important ideas in a proof. Be careful, however, that you do not rely on pictures to the detriment of logic. Always ask yourself if there are other possible pictures and if your argument relies too heavily on the particular picture you have drawn.

Final Project and Presentation: Each student will do a small research project on some aspect of graph theory, will write a short paper, and will give an oral presentation on their topic to the entire

class. More details will be distributed later. The presentations will occur during the last week of the semester.

Participation: I expect that you will volunteer answers to questions posed to the class as a whole and that you will be prepared to make a contribution to the discussion if specifically called upon. In the event that I call on you specifically, you do not necessarily need to be able to answer the question I ask, but you should be prepared to give a partial answer, an intelligent guess, or to ask a relevant question. I expect that you will have attempted each day's homework prior to class.

Colloquium: You are encouraged to attend the departmental colloquium whenever it occurs (typically Mondays at 4 PM). *But, you must attend two Mathematics/Statistics Colloquia.* For each you need to write a page summarizing and reacting to the lecture. Attending colloquia helps you develop your own presentation skills and exposes you to the larger context of mathematical endeavors.

Exams: There will be a take-home midterm exams October 9 - 13 (due on the 13th in class) and November 8 - 15 (due on the 15th in class). The final exam will also be take-home and will be due by Saturday, December 16 at 4:30 PM.

How to succeed in this course:

Graph Theory is an intense 300-level mathematics course. The course, therefore, expects a level of sophistication and mathematical maturity beyond that expected by introductory calculus courses. In particular: you are responsible for your own learning and the professor is responsible for assisting you in your quest. You need to be willing to do computations and you need to be willing to wrestle with ideas. The following advice is meant to provide suggestions on get more out of the class and to improve your grades:

(1) *Do some math everyday.* Really.

Learning mathematics is a lot like learning to play an instrument, play a sport, or learn a language. You must practice everyday. The daily homework problems are intended to help you in your daily practice. But you should spend additional time each day working on the weekly homework, reading the text, and studying previous material. On average, you should spend 2 - 3 hours studying for each hour spent in class. In our case, that's 6-9 hours per week of homework and studying.

(2) *Participate in class.* Yes, you.

Asking and answering questions is a great way to stay engaged with the material and verify for yourself that you know what's going on. The more people that participate, the more fun class is. I, and the rest of the class, value your questions and your answers, right or wrong. In fact, giving a wrong answer to a questions is a great way to learn the right answer. Make an effort to connect each class's activities to previous classes. Try to predict where the material will be going in the future. Take good notes and listen. If you can't do both, make a deal with a buddy: you take notes and they listen one day and the next day you switch.

(3) *Read the textbook.*

The lectures and the textbook will often present slightly different views on the same material. The examples in the text will (for the most part) differ from the example presented in

class. As you read the text, relate what the text does to what happened in class: where is it the same, where is it different? If it differs, which approach do you like better what are the pros and cons of each?

(4) *Form a study group.*

Introduce yourself to other people in the class and meet up outside of class to study and work on homework. Be sure that you don't copy answers, but learn from each other and then write the answers on your own. Compare lecture notes to be sure you copied everything correctly. Ask each other questions and explain material to each other. Make the 2nd floor of Davis a happening place.

(5) *Write your own problems.*

As you study for the exams, look back at all the examples done in class, in the text, or on homework. Try modifying them to make up your own problems. Try to solve your own problems what makes your problems easier or more difficult than the ones you've seen before? Feel free to show me what you've done.

(6) *Be curious.* We will be spending a lot of time, exploring the theoretical mathematical ideas. Be curious about why graph theory works the way it does. Ask "why". Try to find applications of what we're learning. Try to invent your own geometry. Schedule time for curiosity. Block off 2 hours every day for graph theory. Any time not spent working on homework, spend studying and playing with geometric concepts. Come up with your own research questions. Computers compute; humans ask creative questions.

(7) *Visit me.*

I love working with students and I love to help you understand and appreciate the beautiful world of mathematics. Feel free to drop by, even when it's not my office hours. If I can't chat, I'll let you know. Ask crazy questions about the course. Ask questions about my research. Tell me about your past math experiences. Tell me about what subjects you love. Let me know when you have a concert or athletic event. Colby has a take-your-professor to lunch program where I get free lunch if you sign me in. Let's have lunch!

(8) *Spread the studying out over the semester.*

If you do math everyday, as suggested above, you won't have to cram for exams. You'll be able to sleep and, consequently, to think. You'll be happier and more relaxed. You'll have time to write papers for your other classes. You'll have time to appreciate the snow and the spring. You don't need to pull all-nighters.