Fall 2017/MA 397 **HW 6: Connectedness and Flows**

Remember that, although you are encouraged to work together, all of your write-ups must be your own (no copying someone else's solution - not even with minor wording changes.) **List the names of everyone you worked with on the HW!** You are encouraged to **not** look online for solutions - your time is better spent wrestling with the proof yourself or getting help from the professor, than squandering it online.

1. Reading

Read sections 4.1, 4.2, and 4.3. Focus on the section on Blocks, Theorem 4.2.2, Theorem 4.2.4, Theorem 4.2.17 (Menger's Theorem), and Theorem 4.3.11.

Memorize the definition of **vertex cut**, k**-connected**, **connectivity** k, **depth first search**, **network**, **flow**, **capacity constraints**, **conservation constraints**, **max flow/min cut**

2. Problems

Do

(1) Exercise 4.1.8

(2) Exercise 4.1.12

Hint: Label the vertices $v_0, \ldots, v_{n-1}$. Call the edges $v_i v_{i+(n/2)}$ joining v_i to the opposite vertex "special edges". For vertices u, and v let $C(u,v)$ denote the set of vertices encountered when moving clockwise around the circle (not including u and v). Suppose S is a separating set. Then there exist vertices x and y such that every path from x to y passes through S. Show that $|S| \geq k$ unless S contains exactly f consecutive vertices from each of $C(x,y)$ and $C(y,x)$. Now construct an xy-path using a special edges from x or y.

(3) Exercise 4.2.20

Hint: Induct on k. Let x and y be vertices in Q_{k+1}. If they agree in some coordinate, apply the induction hypothesis to the copy of Q_k corresponding to that coordinate. Construct one additional path beyond what the induction hypothesis provides. If they don't agree in any coordinate, define paths explicitly by specifying what order to toggle the entries of x to obtain the entries of y.

(4) Exercise 4.3.1