Fall 2017/MA 397 Final Exam Study Guide

The exam will consist of definitions, calculations of basic graph quantities, proofs of theorems covered in class/reading/HW, and proofs of some new theorems. These new theorems may be very hard, in which case my goal is simply to see how well you can figure out an approach to proving the theorem.

1. Definitions

Know precise definitions of the following terms:

- graph, simple graph, digraph, network, tournament
- graph complement, clique, independent set, bipartite set
- path, cycle, Petersen graph, induced subgraph
- walk, trail, Eulerian circuit
- graph isomorphism
- cut edge, cut vertex
- planar graph
- chromatic number, girth
- degree of a vertex, k-regular graph, neighborhood of a vertex
- acyclic graph, tree, leaf, forest, distance, diameter, spanning tree, Wiener index, Prüfer code
- matching, vertex cover, stable matching
- vertex cut, k-connected, connectivity k, flow
- proper coloring, chromatic number, chromatic polynomial, cartesian product of graphs
- planar graph

You should be able to give precise statements of the following theorems. Unless they also appear below you do not need to know their proofs.

- Max-flow/Min-Cut Theorem
- Gale-Shapely Stable Matching Theorem
- Four-Color Theorem
- Kuratowski's Theorem

2. Calculations

You should remember the steps of the following algorithms and be able to implement them by hand on small examples.

- Kruskal's algorithm for finding a minimum weight spanning tree
- Breadth First Search

- Dijkstra's algorithm for finding minimum distance between two vertices
- Augmenting Path Algorithm (3.2.1)
- Hungarian Algorithm (3.2.9)
- Gale-Shapely Proposal Algorithm
- Ford-Fulkerson labeling algorithm (4.3.9)

You should be able to compute the following quantities/objects for particular graphs. For the algorithms, unless specified below, you do not need to be able to prove the correctness of the algorithm.

- Diameter
- Distance between two vertices in a graph
- maximal size of a clique, maximal size of an independent set.
- max degree and min degree
- chromatic number of a graph
- chromatic polynomial of a graph
- connectivity of a graph
- Wiener index
- Prüfer code given a tree
- Tree given a Prüfer code
- Is a graph eulerian? If so, find a cycle decomposition and an euler circuit.
- Is a digraph eulerian? If so, find a cycle decomposition and an euler circuit.
- Find a spanning tree in a connected graph
- Count the number of spanning trees in a graph using Proposition 2.2.8.
- Finding a proper coloring using the greedy algorithm.
- Mycielski's construction.
- Computing the chromatic polynomial recursively (5.3.6)

3. Previously studied results

- (König's theorem) A graph is bipartite if and only if it has no odd cycle.
- Degree-sum formula
- If G is loopless, there exists a bipartite subgraph with at least $e(G)/2$ edges. (Theorem 1.3.19)
- Connectivity of the cube Q_n is n.
- Among trees with n vertices, the Wiener index is uniquely minimized by stars and uniquely maximized by paths. (2.1.14)
- Cayley's Formula (Theorem 2.2.3)

- Berge's Theorem on Maximum matching (3.1.10) (including 3.1.9)
- Hall's Theorem (3.1.11)
- König-Egerváry Theorem on maximum matchings/minimum vertex covers in bipartite graphs (3.1.16)
- Whitney's theorem on 2-connectedness and disjoint u, v-paths (4.2.2)
- Menger's Theorem on cuts and internally disjoint paths (4.2.17) **outline only**
- Max flow/Min cut Theorem with rational capacities (4.3.11)
- Gale's Theorem on feasible flows in transportation networks (4.3.17)
- Mycielski's construction produces a $k+1$ chromatic triangle free graph from a k chromatic triangle free graph.
- The chromatic polynomial is a polynomial (5.3.4)
- Among smiple r-colorable graphs with n vertices, the Turán graph $T_{n,r}$ is the unique graph with the most number of edges.
- Every planar graph has a vertex of degree ≤ 5.
- 5-color and 6-color theorems for planar graphs
- Kuratowski's Theorem **outline** only.