## Fall 2016/MA 314 Reading Assignment 4

Typically reading assignments expect only a superficial understanding of the material. They are intended to give you a first look at material we'll explore in more depth in class. Occasionally, however, you will be asked to thoroughly understand something from the reading which we will not go through in class.

Most of the readings are from Bonahon, some are from Schwartz and sometimes you are asked to read online materials. Answer the following questions and turn in on the due date. The questions are intended to help you focus on what you are expected to get out of the reading and to ensure you are prepared for class. Your answers need not be long, but they should be thoughtful.

- (1) Bonahon: Chapter 2.1 2.5. Also look at appendix T.4 on complex numbers, if necessary. Pay particular attention to the proof of Lemma 2.1.
  - (a) What is the hyperbolic plane? How do you measure the length of a path in it?
  - (b) Outline the key steps for showing that the hyperbolic metric satisfies the triangle inequality.
  - (c) How is the proof that the hyperbolic metric is an actual metric, and not just a semi-metric similar to the proof that a euclidean line segment is the shortest path between two points in euclidean space?
  - (d) List some isometries of  $\mathbb{H}^2$ .
  - (e) What are some examples of geodesics in  $\mathbb{H}^2$ ?
  - (f) What is a linear fractional map?
  - (g) What is the Riemann sphere?
- (2) Schwartz: Chapter 10.1 10.6. Focus on 10.1 and 10.2.
  - (a) Outline the proof that linear fractional transformations preserve circles.
  - (b) Notice that Schwartz also proves that real linear fractional transformations are hyperbolic isometries. He does not however prove that all hyperbolic isometries can be represented by such a transformation.
  - (c) Is Schwartz's classification (Theorem 10.8) of geodesics of H<sup>2</sup> much different from Bonahon's? Why or why not?
  - (d) Whose presentation do you prefer Bonahon's or Schwartz's?
- (3) Gary Antonick's blog post on hyperbolic geometry "The Non-Euclidean Geometry of Whales": http://wordplay.blogs.nytimes.com/2012/10/08/whale/

(The purpose of this blog reading is twofold: First, you get to read an example of a well-written blog essay on a geometric topic – should be helpful for when you have to write your own! Second, you get to learn some interesting facts about geodesics, a central concept of this course. The questions below are intended to have you explicitly consider the qualities of good mathematical writing.)

- (a) Share some of the interesting things you learned from the essay.
- (b) What questions does the essay provoke? What would you be interested in pursuing further?
- (c) Who is the intended audience of the essay? What qualities of the writing make the author successful or unsuccessful at reaching that audience.

- (d) Choose two other possible audiences for a blog post on the same topic as the essay you read. Say what those audiences are and, for each, how the essay might be structured differently in tone and content if it were written for that audience.
- (4) Watch the video on Möbius transformations by Douglas Arnold and Jonathan Rogness.

(Warning: This essay gets rather technical. Do not feel like you need to understand the technicalities.)

- (a) Share some of the interesting things you learned from the essay.
- (b) What questions does the essay provoke? What would you be interested in pursuing further?
- (c) Who is the intended audience of the essay? What qualities of the writing make the author successful or unsuccessful at reaching that audience.
- (d) Choose two other possible audiences for a blog post on the same topic as the essay you read. Say what those audiences are and, for each, how the essay might be structured differently in tone and content if it were written for that audience.