**Fall 2016/MA 314** **HW 5: The shortest path not taken.**

## 1. Reading

Read Bonahon Chapters 4 and 5 (again!) and Schwartz 12.1 - 1.5.

## 2. To do

(1) Let $X$ be the strip in $\mathbb{R}^2$ bounded by the lines $x = -1$ and $x = +1$. For a fixed $r \geq 0$, let $\overline{X}_r$ be the result gluing the point $(-1, y)$ to $(1, y + r)$ for each $y \in \mathbb{R}$. We can give $\overline{X}_r$ a path metric inherited from $X$ as discussed in class for the torus. For the following problems, you may work informally. Recall that a geodesic is a locally length-minimizing path. The geodesics in $\overline{X}_r$ are all unions of straight line segments in $X$.

   (a) For $r = 0$, sketch a picture of a geodesic loop in $\overline{X}_0$. What is its length? Are there any other geodesic loops?

   (b) For $r = 1$, sketch a picture of a geodesic loop in $\overline{X}_1$. What is its length? Are there any other geodesesic loops?

   (c) Prove that if $r, s \geq 0$ with $r \neq s$, then $\overline{X}_r$ is not isometric to $\overline{X}_s$.

   (d) Describe the isometries of $\overline{X}_r$ for different values of $r$. Do the $\overline{X}_r$ for different values of $r$ all have similar types of isometries?

(2) (extra-credit) Bonahon Exercises 4.7 and 4.8